Reply to Nespolo's paper entitled "New invariants and dimensionless numbers: Futile renaissance of old fallacies?"

Dear Editor:

A recent edition of Biological Research (38: 27-29, 2005) included an article by R. F. Nespolo with a very unusual title: "*New invariants and dimensionless numbers: futile renaissance of old fallacies?*" This paper contains a rather ranting criticism of our paper "*Homeostasis and heterostasis: from invariant to dimensionless numbers*" (Biol. Res. 36: 211-221, 2003), and therefore we are obliged to respond to Nespolo's comments in detail.

A) GENERAL OBJECTIONS

Our disagreements with Nespolo's commentary begin with the title. What are the so-called "old fallacies" that he refers to? He fails to clearly identify them. Furthermore, we believe that the concept of "new invariant numbers" does not infer a "renaissance of old fallacies," but rather the addition of new aspects of current research.

It is commonly known that dimensionless and invariant numbers can be obtained by several procedures (Günther and Morgado, 2003a; Li, 2000; Schepartz, 1980; Stahl, 1962) that finally yield "pure" numbers. Moreover, an invariant number simply means devoid of physical dimensions ($M^0L^0T^0$), while dimensionless numbers can maintain a significant dependence on body mass, a fact that can be exemplified by "residual mass exponents." From this perspective, an invariant number should be a constant, although in Charnov's words, "how constant is constant enough to be considered invariant is worthy of much thought" (1993, p: 5).

As is well known, some "invariant" numbers can be obtained by combining universal constants, although the numerical values of these universal constants are experimentally determined and subject to statistical variations. The different values obtained for these universal constants can lead to significant contributions in cosmology, such as the prediction of the existence of innumerable parallel universes (Barrow and Webb, 2005). From this perspective, the matter of dimensionless or invariant numbers is not a "fallacy," and therefore "not futile."

The subject of dimensionless and invariant numbers is currently an active field of investigation, and certainly not restricted to physics. For example, the "Handbook of Physics and Chemistry," features several hundred dimensionless groups with applications in modern engineering (Weast, 1983). The biological sciences also include many contributions in this field (Vogel, 2003; Charnov, 1993; Brown and West, 2000).

When Buckhingham's theorem is used to calculate dimensionless numbers from a set of dimensional variables, dimensionless parameters that are independent of body mass are obtained first and their ratio is not only dimensionless, but also independent of body mass. In this sense, they are considered invariant numbers as well as dimensionless constants. This invariance means that the relationship obtained is valid for the entire animal model. Some authors have also employed allometric equations to obtain dimensionless and invariant numbers (Li, 2000).

The dimensionless and invariant numbers mentioned in our article (Günther et al., 2003b) were calculated by means of three different procedures, and not based simply on allometric equations (for other examples, see Günther and Morgado, 2002). The dimensionless and invariant numbers obtained from the allometric equations were only used as a reference, and these entities were calculated from actual data. Since the data is very scant, we used data from one of our previous papers (Barger et al., 1956) to avoid introducing pooled data from different sources in our calculations. It must be remembered that some experiments are unique (N = 1) and that small figures do not invalidate the conclusions

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(for example, one artificial satellite recently collided with one comet).

B) SPECIFIC OBJECTIONS.

Nespolo's equation (2) is an erroneous transcription of our equation (2):

Nespolo's equation (2) reads:
$W^b = (W^{0.023}(W^{0.07})^3(W^{0.27})^3/ W^{0.73}W^{-0.68}Q^{0.99})$

Observe also that Nespolo's paragraph following equation (2) refers to variables cited in the text (P, V, T, B, Tp), which do not correspond to the equation mentioned. From our Table I, $Q = W^{0.99}$, and introducing Q raised to 0.99 makes the entire equation erroneous and non-homogeneous from a dimensional perspective.

Thus, the correct equation is as follows:

$$(2)\quad IN_1 = \frac{P \cdot (\bar{v})^3 \cdot T^3}{\dot{V}_{O_2} \cdot TPR \cdot Q_b} =$$

$$\frac{5.8 \times 10^4}{1.95 \times 10^2} = 2.989 \times 10^2 \approx 300$$

The variables, their units, and the corresponding allometric equations (parameters and exponents) are presented in our Table I. The replacement of the variables by the corresponding allometric equation leads to:

$$(3)\quad W^b = \frac{W^{0.023}\ (W^{0.07})^3\ (W^{0.27})^3}{W^{0.73}\ W^{-0.68}\ W^{0.99}}$$

The resolution of equation (3) using the corresponding parameters and W raised to the proper exponents gives:

$$(4)\quad IN_1 = 300W^{\,0.003}$$

The most relevant part of our reasoning, the reaching of a dimensionless number (300) is missed in Nespolo's erroneous interpretation. Observe that the residual mass exponent is 0.003 and not 0.03 (Nespolo 2005, page 28, final paragraph).

The dimensionless number can be obtained from the units of the variables involved:

P is expressed as (dynes · cm^{-2});

V^3 is expressed as ($cm^3 \cdot s^{-3}$);

T^3 is expressed as (s^3);

V_{O2} is expressed as ($cm^3 \cdot s^{-1}$)

TPR is expressed as (dynes · s · cm^{-5}); and

Q is expressed as (cm^3)

The numerator of IN1 is: (dynes · cm^{-2}) ·($cm^3 \cdot s^{-3}$) · (s^3) = (dynes · cm)

The denominator of IN1 is: ($cm^3 \cdot s^{-1}$) · (dynes · s · cm^{-5}) · (cm^3) = (dynes · cm), and in consequence, a dimensionless number is obtained. Please observe that the use of allometric equations (which may obviously have statistical pitfalls) is not necessary to obtain the dimensionless number.

Thus, the dimensionless number obtained from allometric equations is only used as a reference, since we employ ***actual*** data obtained from the same experimental animal to calculate the dimensionless number in resting and exercise conditions. We have not "replaced those values in the allometric equation," as Nespolo suggests (Nespolo 2005, page 29, first paragraph). The data was replaced in IN1.

We do agree that the number of animals is very low, but it was impossible to obtain a relevant number of cases from the literature. The great majority of textbooks only provide the values of some of the variables involved in resting and exercise conditions, primarily in humans, without indicating the corresponding body weights, number of cases, or statistics and dispersion measures. In any case, our dog behaves as has been described in the literature (Skinner, 1991).

Nespolo lists five reasons to "find this slight variation of the invariant number purely due to methodological and scaling artifacts" (p: 28). We do not use the phrase "variation of the invariant number" in any part of our paper.

Nespolo's first reason is disputable. In biology it is very difficult to define a number as large or small or to determine whether it does or does not pertain to a given group. (Norman and Streiner, 1994). The ratio of IN1 between resting and maximal exercise represents a 3% variation. The 95% confidence limits of the entire data set are 142 and 205. The 95% confidence limits of the exercising dog are 112 and 232. The resting value of IN1 is 176.5, which is included within the 95% confidence limits of the exercising dog. The same occurs when the data is transformed into logarithms (the 95% confidence limits for the logarithmic transformation are 119 and 246).

The second reason is widely known: ratios and logarithms make the numbers small. However, our residual mass exponent is 0.003 and not 0.03. Furthermore, we are not using allometric equations to calculate IN1.

The third reason is partially valid. We do agree, in fact, that the number of cases in our example is small (N = 1), but we are not replacing the actual data in allometric equations. It is also interesting to bear in mind that Wunderlich measured nearly 1,000,000 axilar temperatures in 25,000 subjects and determined 37ºC to be the "normal" temperature. Modern measures revealed that Wunderlich's thermometers were erroneously calibrated (Mackowiak et al., 1992).

Nespolo's fourth reason is invalid for the simple reason that we do not compare the dimensionless number obtained from actual data with the dimensionless number obtained from allometric equations.

The fifth reason is also disputable. We do not deny that the variables are in some way connected (in physiology it is very difficult to identify two variables that are not correlated). Our Figure 1 emphasizes the reticular structure of the circulation. Obviously, to obtain the same number under two different conditions it is necessary to have certain variables increase while others decrease.

It is quite curious that Nespolo does not criticize our DN2 in his article. In this case, the dimensionless number obtained from the allometric equations is 824. The number obtained from data under shock conditions (obviously not experimental data) is 37.5. The ratio of the two is near 22. This figure seems to be large enough for Nespolo, but it was obtained from allometric equations and from pooled data not pertaining to the same individual.

Nespolo concludes by tritely reminding the reader of basic scientific and philosophic definitions and making a number of negative assertions about our article when in fact his claims themselves are based on an erroneous interpretation of our work, and therefore moot.

In sum, Nespolo's "old fallacies" article deserves an old Shakespearean title: "Much Ado About Nothing."

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