Erratum to “on the group of strong symplectic homeomorphisms”

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ABSTRACT

We give a proof of the estimate (1.1) which is the main ingredient in the proof that the set $SSympeo(M, \omega)$ of strong symplectic homeomorphisms of a compact symplectic manifold $(M, \omega)$ forms a group [1].

RESUMEN

Probamos la estimación (1.1) que es el principal elemento en la demostración que el conjunto $SSympeo(M, \omega)$ de homeomorfismos simplécticos fuertes de una variedad simpléctica compacta $(M, \omega)$ genera un grupo [1].

Keywords and Phrases: $C^0$-symplectic topology; Strong symplectic homeomorphism.

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1 Erratum

In the paper [1] mentioned in the title, the “constant $E$” at page 60 may be infinite (so proposition 2 is meaningless). Therefore, some of the estimates on pages 63 to 65 based on $E$, needed to show that

$$\int_0^1 \text{osc}(v^n_t - v^m_t) \to 0 \quad \text{(1.1)}$$

as $n, m \to \infty$ may not hold true. Here is a direct proof of [1].

First simplify the notations by writing $kH^m_t$ for $\left(\mu^k_t\right)^*H^m_t$, $H$ for $H$ and omitting $t$. The function $v^n := v^n_t$ satisfies $nH^n_t - H^n = dv^n$. Fix a point $\ast$ in $M$ and for each $x \in M$, pick an arbitrary curve $\gamma_x$ from $\ast$ to $x$, then

$$u^n(x) := \int_{\gamma_x} (nH^n_t - H^n) = v^n(x) - v^n(\ast).$$

(The definition of $u^n(x)$ is independent of the choice of the curve $\gamma_x$). Hence $\text{osc}(u^n - u^m) = \text{osc}(v^n - v^m)$. Since $\text{osc}(f) \leq 2|f|$, where $|.|$ is the uniform sup norm, we need to show that

$$\int_0^1 \left| \int_{\gamma_x} (nH^n_t - H^n) - (mH^m_t - H^m) \right| dt \leq \int_0^1 \left| \int_{\gamma_x} (nH^n_t - mH^m_t) \right| dt$$

$$+ \int_0^1 \left| \int_{\gamma_x} (H^n_t - H^m_t) \right| dt, \quad \text{(1.2)}$$

goes to zero, when $n, m$ are sufficiently large.

The last integral tends to zero when $n, m$ are large: indeed,

$$\int_0^1 \left| \int_{\gamma_x} (H^n_t - H^m_t) \right| dt = \int_0^1 \left| \int_0^1 (H^n_t - H^m_t)(\gamma_x(u))(\gamma_x'(u)du) \right| dt$$

$$\leq A \int_0^1 |H^n - H^m| dt, \quad \text{(1.3)}$$

where $A = \sup_u |\gamma_x'(u)|$. This goes to 0 since $H^n$ is a Cauchy sequence.

To prove that $\int_0^1 \left| \int_{\gamma_x} (nH^n_t - mH^m_t) \right| dt$ tends to zero when $n, m \to \infty$, we write:

$$\left| \int_{\gamma_x} (nH^n_t - mH^m_t) \right| \leq \left| \int_{\gamma_x} (nH^n_t - H^n_t) \right|$$

$$+ \left| \int_{\gamma_x} (m(H^n_t - H^m_t) - n_0(H^n - H^m)) \right|$$

$$+ \left| \int_{\gamma_x} (n_0)(H^n - H^m) \right|, \quad \text{(1.4)}$$

for some large $n_0$. 

The integral
\[
\int_0^1 \left| \int_{\gamma_x} (n_0)(H^n - H^m) \right| dt = \int_0^1 \left| \int_0^1 (H^n - H^m)(\gamma_{x_0}(u))(D\mu^{n_0}\gamma'_x(u))du \right| dt \\
\leq B \int_0^1 |H^n - H^m| dt,
\]
where \( B = \sup_u |D\mu^{n_0}\gamma'_x(u)| \) goes to zero when \( n, m \to \infty \) since \( H^n \) is a Cauchy sequence and \( D\mu^{n_0} \) is bounded. (Here \( \gamma_k = \mu^k(\gamma_x) \)).

We now show that \( \int_{\gamma_x} (nH^n - mH^n) = \int_{\gamma_n} H^n - \int_{\gamma_m} H^n \) tends to zero when \( n, m \to \infty \).

Let \( d_0 \) be a distance induced by a Riemannian metric \( g \) and let \( r \) be its injectivity radius. For \( n, m \) large enough, \( \sup_x d_0(\mu^n_t(x), \mu^m_t(x)) \leq r/2 \). It follows that there is a homotopy \( F : [0,1] \times M \to M \) between \( \mu^n_t \) and \( \mu^m_t \), i.e \( F(0,y) = \mu^n(y) \) and \( F(1,y) = \mu^m(y) \) and we may define \( F(s,y) \) to be the unique minimal geodesic \( v_{mn}^s \) joining \( \mu^n(y) \) to \( \mu^m(y) \). See [3] (Theorem 12.9). Let \( \Box(s,u) = \{F(s,\gamma_n(u)), 0 \leq s, u \leq 1\} \).

Since by Stokes' theorem, \( \int_{\partial \Box} H^n = 0 \), \( \int_{\gamma_x} H^n - \int_{\gamma_m} H^n = \int_{L} H^n - \int_{L'} H^n \) where \( L \) and \( L' \) are the geodesics \( v_{mn}^x \) and \( v_{nm}^x \). The integral over \( L \) is bounded by \( \sup_s |H^n(v_{mn}^s)\|d_0(\mu^n_t(x), \mu^m_t(x))| \), because the speed of the geodesics \( L, L' \) is bounded by \( d_0(\mu^n_t(x), \mu^m_t(x)) \). This integral tends to zero when \( n, m \to \infty \) since \( H^n \) is also bounded. Analogously for the integral over \( L' \).

The same argument apply to \( H^n - H^m \) with the geodesics \( L, L' \) replaced by \( v_{mn_0}^x \) and \( v_{nm_0}^x \). This finishes the proof of (1.1).

**Remark** : We will show in a forthcoming paper [2] that (1.1) is the main ingredient in the proof of the main theorem of [1].

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**References**

[1] A. Banyaga, On the group of strong symplectic homeomorphisms, Cubo, Vol 12, No 03 (2010), 49-69

