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A Note on Modifications of $\mathfrak{rg}$-Closed Sets in Topological Spaces

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ABSTRACT

We point out that a certain modification of regular generalized closed sets due to Palaniappan and Rao [15] means nothing to the family of semi-open sets.

RESUMEN

Destacamos que una modificación de conjuntos cerrados regulares generalizados debido a Palaniappan and Rao [15] no tiene importancia para la familia de conjuntos semi-abiertos.

Keywords and Phrases: $\mathfrak{g}$-closed, $\mathfrak{rg}$-closed.

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1 Introduction

In 1970, Levine [11] introduced the notion of generalized closed (briefly g-closed) sets in topological spaces and showed that compactness, locally compactness, countably compactness, paracompactness, and normality etc are all g-closed hereditary. And also he introduced a separation axiom called $T_{1/2}$ between T_1 and T_0. Since then, many modifications of g-closed sets are introduced and investigated. Among them, Dontchev and Ganster [5] introduced the notion of $T_{3/4}$-spaces which are situated between T_1 and $T_{1/2}$ and showed that the digital line or the Khalimsky line [9] $(\mathbf{Z}, \kappa)$ lies between T_1 and $T_{3/4}$.

As a modification of g-closed sets, regular generalized closed sets are introduced and investigated by Palaniappan and Rao [15]. As the further modification of g-closed sets, Gnanambal [7] introduced the notion of generalized preregular closed sets. The purpose of this note is to present some remarks concerning modifications of regular generalized closed sets.

2 Preliminaries

Let (X, τ) be a topological space and A a subset of X. The closure of A and the interior of A are denoted by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. We recall some generalized open sets in topological spaces.

Definition 2.1. Let (X, τ) be a topological space. A subset A of X is said to be

(1) α-*open* [14] if $A \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$,

(2) *semi-open* [10] if $A \subset \mathrm{Cl}(\mathrm{Int}(A))$,

(3) *preopen* [12] if $A \subset \mathrm{Int}(\mathrm{Cl}(A))$,

(4) *semi-preopen* [2] or β-*open* [1] if $A \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$,

(5) *b-open* [3] if $A \subset \mathrm{Int}(\mathrm{Cl}(A)) \cup \mathrm{Cl}(\mathrm{Int}(A))$,

(6) *regular open* if $A = \mathrm{Int}(\mathrm{Cl}(A))$.

The family of all α-open (resp. semi-open, preopen, semi-preopen, b-open, regular open) sets in (X, τ) is denoted by τ^α (resp. $\mathrm{SO}(X)$, $\mathrm{PO}(X)$, $\mathrm{SPO}(X)$, $\mathrm{BO}(X)$, $\mathrm{RO}(X, \tau)$).
For generalizations of open sets defined above, the following relations are well known:

DIAGRAM I

$$\begin{array}{ccccccc} \text{open} & \Rightarrow & \alpha\text{-open} & \Rightarrow & \text{preopen} & & \\ & & \Downarrow & & \Downarrow & & \\ & & \text{semi-open} & \Rightarrow & b\text{-open} & \Rightarrow & \text{semi-preopen} \end{array}$$

Definition 2.2. Let (X, τ) be a topological space. A subset A of X is said to be α-*closed* [13] (resp. *semi-closed* [4], *preclosed* [12], *semi-preclosed* [2], *b-closed* [3]) if the complement of A is α-open (resp. semi-open, preopen, semi-preopen, b-open).

Definition 2.3. Let (X, τ) be a topological space and A a subset of X. The intersection of all α-closed (resp. semi-closed, preclosed, semi-preclosed, b-closed) sets of X containing A is called the *α-closure* [13] (resp. *semi-closure* [4], *preclosure* [6], *semi-preclosure* [2], *b-closure* [3]) of A and is denoted by $\alpha\mathrm{Cl}(A)$ (resp. $\mathrm{sCl}(A)$, $\mathrm{pCl}(A)$, $\mathrm{spCl}(A)$, $\mathrm{bCl}(A)$).

Definition 2.4. Let (X, τ) be a topological space. A subset A of X is said to be

(1) *generalized closed* (briefly *g-closed*) [11] if $\mathrm{Cl}(A) \subset U$ whenever $A \subset U$ and $U \in \tau$,

(2) *regular generalized closed* (briefly rg-*closed*) [15] if $\mathrm{Cl}(A) \subset U$ whenever $A \subset U$ and $U \in \mathrm{RO}(X, \tau)$,

(3) *generalized preregular closed* (briefly gpr-*closed*) [7] if $\mathrm{pCl}(A) \subset U$ whenever $A \subset U$ and $U \in \mathrm{RO}(X, \tau)$.

For generalizations of closed sets defined above, the following relations are well known:

DIAGRAM II

closed $\Rightarrow$ g-closed $\Rightarrow$ rg-closed $\Rightarrow$ gpr-closed

3 Modifications of rg-closed sets

First we shall define a modification of rg-closed sets.

Definition 3.1. Let (X, τ) be a topological space. A subset A of X is said to be *regular generalized α-closed* (briefly $rg\alpha$-*closed*) if $\alpha\mathrm{Cl}(A) \subset U$ whenever $A \subset U$ and $U \in \mathrm{RO}(X, \tau)$.

Lemma 3.2. *If A is a subset of* (X, τ)*, then* $\tau^\alpha\text{-Int}(\tau^\alpha\text{-Cl}(A)) = \mathrm{Int}(\mathrm{Cl}(A))$.

Proof. This is shown in Corollary 2.4 of [8].

Lemma 3.3. *Let V be a subset of a topological space* (X, τ)*. Then* $V \in \mathrm{RO}(X, \tau)$ *if and only if* $V \in \mathrm{RO}(X, \tau^\alpha)$.

Proof. This is an immediate consequence of Lemma 3.2.

Theorem 3.4. *A subset A of a topological space* (X, τ) *is* $rg\alpha$*-closed in* (X, τ) *if and only if A is* rg*-closed in the topological space* (X, τ^α).

Proof. *Necessity.* Suppose that A is $rg\alpha$-closed in (X, τ). Let $A \subset V$ and $V \in \mathrm{RO}(X, \tau^\alpha)$. By Lemma 3.3, $V \in \mathrm{RO}(X, \tau)$ and we have $\tau_\alpha\text{-Cl}(A) = \alpha\mathrm{Cl}(A) \subset V$. Therefore, A is rg-closed in (X, τ^α).

Sufficiency. Suppose that A is rg-closed in (X, τ^α). Let $A \subset V$ and $V \in \mathrm{RO}(X, \tau)$. By Lemma 3.3, $V \in \mathrm{RO}(X, \tau^\alpha)$ and hence $\alpha\mathrm{Cl}(A) = \tau_\alpha\text{-Cl}(A) \subset V$. Therefore, A is $rg\alpha$-closed in (X, τ).

Remark 3.5. It turns out that, by Therem 3.4, we can not obtain the essential notion even if we replace Cl(A) in Definition 2.4(2) with αCl(A).

Next, we try to replace Cl(A) in Definition 2.4(2) with sCl(A).

Lemma 3.6. *Let* (X, τ) *be a topological space. If* $A \subset V$ *and* $V \in RO(X, \tau)$*, then* $sCl(A) \subset V$.

Proof. Let $A \subset V$ and $V \in RO(X, \tau)$. Then we have $sCl(A) \subset sCl(V) = V \cup Int(Cl(V)) = V$ and hence $sCl(A) \subset V$.

Remark 3.7. (1) Lemma 3.6 shows that in case $SO(X, \tau)$ the condition "$sCl(A) \subset V$ whenever $A \subset V$ and $V \in RO(X, \tau)$" does not define a subset like *regualr generalized semi-closed sets*.

(2) By Diagram I, $SO(X) \subset BO(X) \subset SPO(X)$ and hence $spCl(A) \subset bCl(A) \subset sCl(A)$ for any subset A of X. Therefore, we can not obtain any notions even if we replace Cl(A) in Definitiion 2.4(2) with sCl(A), bCl(A) or spCl(A).

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