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Generalization of New Continuous Functions in Topological Spaces

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ABSTRACT

In this paper, $\omega\alpha$-closed sets and $\omega\alpha$-open sets are used to define and investigate the new classes of functions namely somewhat $\omega\alpha$-continuous functions and totally $\omega\alpha$-continuous functions.

RESUMEN

En este artículo conjuntos cerrados-$\omega\alpha$ y abiertos-$\omega\alpha$ se usan para definir e investigar las clases de nuevas funciones continuas $\omega\alpha$ y totalmente continuas $\omega\alpha$.

Keywords and Phrases: $\omega\alpha$-closed, $\omega\alpha$-open, $\omega\alpha$-continuous, somewhat $\omega\alpha$ -continuous and totally $\omega\alpha$- continuous functions.

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1 Introduction

Recent progress in study of charactreization and generalization of continuity has been done by means of several generalized closed sets. As a generalization of closed sets $\omega\alpha$-closed sets were introduced and studied by Benchalli.et.al[1].

The concepts of feebly continuous functions and feebly open functions were introduced by Zdenek Frolik[2]. Gentry and Hoyle[3] introduced and studied the concepts of somewhat continuous functions and somewhat open functions. Recently, Santhileela and Balasubramanian[8] introduced and studied the concepts of somewhat semi continuous functions and somewhat semi open functions. In this paper, we will continue the study of related functions with $\omega\alpha$-closed and $\omega\alpha$-open sets. We introduce and characterize the concept of somewhat $\omega\alpha$-contnuous and totally $\omega\alpha$-continuous functions.

2 Preliminaries

Throughout this paper (X, τ), (Y, σ) and (Z, η)(or simply X,Y and Z) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned.For a subset A of (X, τ), $cl(A)$,$int(A)$, $\alpha cl(A)$ and A^c denote the closure of A, inerior of A, the α-closure of A and the compliment of A in X respecively.

We recall the following definitions, which are usefull in the sequel.Before entering into our work we recall the following definitions from various authors.

Definition 2.1. *A subset A of a topological space (X, τ) is called semi-open [5] (resp. α-open[6]) if $A \subseteq cl(Int(A))$ (resp $A \subseteq Int(cl(Int(A)))$.The compliment of semi-open (resp.α-open) is called semi-closed(resp.α-closed).*

Definition 2.2. *A subset A of a topological space (X, τ) is called $\omega\alpha$-closed [1] if $\alpha cl(A) \subseteq U$ whenever $A \subset U$ and U is ω-open in X. The compliment of $\omega\alpha$-closed set is $\omega\alpha$-open.*

The family of all $\omega\alpha$-closed sets of X is denoted by $\tau^*_{\omega\alpha}$. In [7], we showed that $\tau^*_{\omega\alpha}$ forms a topology on X.

Definition 2.3. *A function $f : (X, \tau) \to (Y, \sigma)$ is is said to be $\omega\alpha$-continuous [7] if the inverse image of every open set in Y is $\omega\alpha$-open in X.*

Definition 2.4. *A function $f : (X, \tau) \to (Y, \sigma)$ is is said to be perfectly $\omega\alpha$-continuous [7] if the inverse image of every $\omega\alpha$ open set in Y is clopen in X.*

Definition 2.5. *A function $f : (X, \tau) \to (Y, \sigma)$ is is said to be somewhat-continuous [3](resp.somewhat semi-continuous[8]) if for $U \in \sigma$ and $f^{-1}(U) \neq \phi$ there exists an open (resp.semi open) set V in X such that $V \neq \phi$ and $V \subseteq f^{-1}(U)$.*

Remark 2.6. *Every somewhat continuous function is somewhat semi continuous but converse need not true in general[8].*

Definition 2.7. *A function* $f : (X, \tau) \to (Y, \sigma)$ *is said to be somewhat-open [3](resp.somewhat semi-open[8]) function provided that for* $U \in \tau$ *and* $U \neq \phi$*, there exists an open (resp.semi open) set* V *in* Y *such that* $V \neq \phi$ *and* $V \subseteq f^{-1}(U)$.

Remark 2.8. *Every somewhat open function is somewhat semi open function but the converse need not be true in general[8].*

3 Somewhat $\omega\alpha$ - Continuous functions

In this section, we introduce a new class of functions called somewhat $\omega\alpha$-continuous functions using $\omega\alpha$-closed sets and obtain some of their characterizations.

Definition 3.1. A *function* $f : (X, \tau) \to (Y, \sigma)$ *is said to be Somewhat* $\omega\alpha$*- continuous if for every open set* U *in* Y *and* $f^{-1}(U) \neq \phi$*, there exists* $\omega\alpha$*-open set* V *in* X *such that* $V \neq \phi$ *and* $V \subseteq f^{-1}(U)$.

Example 3.2. *Let* $X = Y = \{p, q\}$, $\tau = \{X, \phi, \}$ *and* $\sigma = \{X, \phi, \{p\}\}$. *The identity function* $f : (X, \tau) \to (Y, \sigma)$ *is somewhat* $\omega\alpha$*-continuous function.*

Theorem 3.3. *Every somewhat continuous function is somewhat* $\omega\alpha$*- continuous but converse need not true in general.*

Example 3.4. *In Example 3.2, f is somewhat* $\omega\alpha$*-continuous but not somewhat continuous.*

Remark 3.5. *The concept of somewhat* $\omega\alpha$*-continuous and somewhat semi-continuous functions are independet as seen from the following examples.*

Example 3.6. *In Example 3.2,f is somewhat* $\omega\alpha$*-continuous but not somewhat-semi continuous.*

Example 3.7. *Let* $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a, b\}\}$ *and* $\sigma = \{X, \phi, \{a\}\}$. *Then the identity map* $f : (X, \tau) \to (Y, \sigma)$ *is somewhat-semi continuous but not somewhat* $\omega\alpha$*-continuous.*

Theorem 3.8. *If* $f : (X, \tau) \to (Y, \sigma)$ *is somewhat* $\omega\alpha$*-continuous and* $g : (Y, \sigma) \to (Z, \eta)$ *is continuous function,then their composition* gof *is somewhat* $\omega\alpha$*-continuous function.*

Proof. Let U be an open set in Z.Suppose that $f^{-1}(U) \neq \phi$. Since U is open and g is continuous, $g^{-1}(U) \in \eta$. Suppose that $f^{-1}(g^{-1}(U)) \neq \phi$. By hypothesis, there exists a $\omega\alpha$-open set V in Y such that $V \neq \phi$ and $V \subseteq f^{-1}(g^{-1}(U)) = (gof)^{-1}(V)$. Therefore gof is somewhat $\omega\alpha$-continuous function. □

Remark 3.9. *In the above Theorem 3.8, if* f *is continuous and* g *is somewhat* $\omega\alpha$*-continuous then their composition* gof *need not be somewhat* $\omega\alpha$*-continuous function as seen from the following example.*

Example 3.10. *Let* $X = Y = Z = \{p, q\}$, $\tau = \{X, \phi, \{p\}\}$, $\sigma = \{Y, \phi, \{p\}\}$ *and* $\eta = \{Z, \phi, \{q\}\}$ *Define the functions* $f : (X, \tau) \to (Y, \sigma)$ *by* $f(p) = f(q) = q$ *and* $g : (Y, \sigma) \to (Z, \eta)$ *by* $g(p) = q$ *and* $g(q) = p$.*Then clearly* f *is continuous function and* g *is somewhat* $\omega\alpha$*-continuous function but their comoposition* $gof : (X, \tau) \to (Z, \eta)$ *is not somewhat* $\omega\alpha$*-continuous function.*

Definition 3.11. A *subset* M *of a topological space* X *is said to be* $\omega\alpha$*-dense in* X *if there is no proper* $\omega\alpha$*-closed set* F *in X such that* $M \subset F \subset X$.

Theorem 3.12. *The following statements are equivalent for a function* $f : (X, \tau) \to (Y, \sigma)$*:*

(1) f *is somewhat* $\omega\alpha$*-continuous function*

(2) *If* F *is a closed subset of* Y *such that* $f^{-1}(F) \neq X$,*then there is a proper* $\omega\alpha$*-closed subset* D *of* X *such that* $f^{-1}(F) \subset D$.

(3) *If* M *is a* $\omega\alpha$*-dense subset of* X*, then* $f(M)$ *is a dense subset of* Y.

Proof. $(1) \Rightarrow (2)$: Let F be a closed subset of Y such that $f^{-1}(F) \neq X$.Then $f^{-1}(Y - F) = X - f^{-1}(f) \neq \phi$. Then from (1) there exists $\omega\alpha$-open set V in X such that $V \neq \phi$ and $V \subset f^{-1}(Y - F) = X - f^{-1}(F)$.This implies $f^{-1}(F) \subset X - V$ and $X - V = D$ is a $\omega\alpha$-closed set inX.
$(2) \Rightarrow (3)$: Let M be any $\omega\alpha$-dense set in X. Suppose $f(M)$ is not a dense subset of Y, then there exists a proper closed set F in Y such that $f(M) \subset F \subset Y$. This implies $f^{-1}(F) \neq X$. Then from (2) there exists a proper $\omega\alpha$-closed set D such that $M \subset f^{-1}(F) \subset D \subset X$. This contradicts the fact that M is a $\omega\alpha$-dense set in X.
$(3) \Rightarrow (2)$: Suppose (2) is not true.Then there exists a closed set$F$ in Y such that $f^{-1}(F) \neq X$.But there is no proper $\omega\alpha$-closed set D in X such that $f^{-1}(F) \subseteq D$. This means that $f^{-1}(F)$ is $\omega\alpha$-dense in X. But from hypothesis $f(f^{-1}(F)) = F$ must be dense in Y, which is contradiction to the choice of F.
$(2) \Rightarrow (1)$:Let U be an open set in Y and $f^{-1}(U) \neq \phi$. Then $f^{-1}(Y - U) = X - f^{-1}(U) = \phi$. Then by hypothesis, there exists a proper $\omega\alpha$-closed set D such that $f^{-1}(Y - U) \subset D$. This implies that $X - D \subset f^{-1}(U)$ and $X - D$ is $\omega\alpha$-open and $X - D \neq \phi$. □

Theorem 3.13. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be a function and* $X = A \cup B$*,* A *and* B *are open subsets of* X *such that* (f/A) *and* (f/B) *are somewhat* $\omega\alpha$*-continuous functions then f is somewhat* $\omega\alpha$*-continuous function.*

Proof. Let U be an open set in Y such that$f^{-1}(U) \neq \phi$. Then $(f/A)^{-1}(U) \neq \phi$ or $(f/B)^{-1}(U) \neq \phi$ or both $(f/A)^{-1}(U) \neq \phi$ and $(f/B)^{-1}(U) \neq \phi$.
case(i): Suppose $(f/A)^{-1}(U) \neq \phi$. Since f/A is somewhat $\omega\alpha$-continuous , then there exists $\omega\alpha$ open set V in A such that $V \neq \phi$ and $V \subset (f/A)^{-1}(U) \subset f^{-1}(U)$. Since V is $\omega\alpha$-open in A and A is open in X, V is $\omega\alpha$-open X . Hence f is somewhat $\omega\alpha$-continuous function.
case(ii): Suppose $(f/B)^{-1}(U) \neq \phi$. Since f/B is somewhat $\omega\alpha$-continuous , then there exists $\omega\alpha$ open set V in B such that $V \neq \phi$ and $V \subset (f/B)^{-1}(U) \subset f^{-1}(U)$. Since V is $\omega\alpha$-open in B and B

is open in X, V is $\omega\alpha$-open X . Hence f is somewhat $\omega\alpha$-continuous function.
case(iii): Suppose $(f/A)^{-1}(U) \neq \phi$ and $(f/B)^{-1}(U) \neq \phi$. Follows from case(i) and case(ii). □

Theorem 3.14. *If A be any set in X and $f : (X, \tau) \to (Y, \sigma)$ be somewhat $\omega\alpha$-continuous such that $f(A)$ is dense in Y. Then any extension F of f is somewhat $\omega\alpha$-continuous.*

Proof. Let U be an open set in$Y$ such that$F^{-1}(U) \neq \phi$. Since $f(A) \subset Y$ is dense in Y and $U \cap f(A) \neq \phi$. It follows that $F^{-1}(U) \cap A \neq \phi$. That is $f^{-1}(U) \cap A \neq \phi$.Hence by hypothesis there exists a $\omega\alpha$-open set V in A such that $V \neq \phi$ and $V \subset f^{-1}(U) \subset F^{-1}(U)$.This implies F is somewhat $\omega\alpha$-continuous. □

Definition 3.15. A *topological space* X *is said to be $\omega\alpha$-separable if there exists a countable subset* B *of* X *which is $\omega\alpha$-dense in* X.

Theorem 3.16. *Let $f : (X, \tau) \to (Y, \sigma)$ is somewhat $\omega\alpha$-continuous function.If X is $\omega\alpha$-separable then Y is separable.*

Proof. Let B be countable subset of X which is $\omega\alpha$-dense in X. Then from Theorem 3.12,$f(B)$ is dense in Y. Since B is countable $f(B)$ is also countable which is dense in Y. This implies that Y is separable. □

4 Somewhat $\omega\alpha$-Open Functions

In this section, we introduce the concept of somewhat $\omega\alpha$-open functions and study some of their characterizations.

Definition 4.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is somewhat $\omega\alpha$-open provided that for open set U in X and $U \neq \phi$ there exists a $\omega\alpha$ -open set V in Y such that $V \neq \phi$ and $V \subseteq f(U)$.*

Example 4.2. *Let $X = Y = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b, c\}\}$ and $\sigma = \{X, \phi, \{a\}\}$. Define a function $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then clearly f is somewhat $\omega\alpha$-open.*

Theorem 4.3. *Every somewhat open function is somewhat $\omega\alpha$-open function but converse need not be true in general.*

Example 4.4. *In Example 4.2, f is somewhat $\omega\alpha$-open function but not somewhat -open function.*

Remark 4.5. *Somewhat $\omega\alpha$-open and somewhat semi-open functions are independent of each other as seen from the following examples.*

Example 4.6. *In Example 4.2, f is somewhat $\omega\alpha$-open function but not somewhat semi-open function.*

Example 4.7. *Let* $X = Y = \{a, b, c\}$*,* $\tau = \{X, \phi, \{b\}, \{a, c\}\}$ *and* $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}\}$*. Then the identity function* $f : (X, \tau) \to (Y, \sigma)$ *is somewhat semi-open but not somewhat* $\omega\alpha$*-open function.*

Theorem 4.8. *If* $f : (X, \tau) \to (Y, \sigma)$ *is open function and* $g : (Y, \sigma) \to (Z.\eta)$ *is somewhat* $\omega\alpha$*-open function,then their composition* gof *is somewhat* $\omega\alpha$*-open function.*

We have the following characterization.

Theorem 4.9. *The following statements are eqivalent for bijective function* $f : (X, \tau) \to (Y, \sigma)$

(1) *f is somewhat* $\omega\alpha$*-open function*

(2) *If* F *is closed subset of* X *such that* $f(F) \neq Y$*, then there exists a* $\omega\alpha$*-closed subset* D *of* Y *such that* $D \neq Y$ *and* $f(F) \subset D$*.*

Proof. $(1) \Rightarrow (2)$:Let F be a closed subset of X such that $f(F) \neq Y$. From (1), there exists a $\omega\alpha$ -open set $V \neq \phi$ in Y such that $V \subset f(X - F)$. Put $D = Y - V$. Clearly D is a $\omega\alpha$-closed in Y and we claim that $D \neq Y$. If $D = Y$, then $V = \phi$ which is a contradiction. Since $V \subset f(X - F)$, $D = Y - V \subset Y - [f(X - F)] = f(F)$.
$(2) \Rightarrow (1)$:Let U be any non-empty open set in X. Put $F = X - U$. Then F is a closed subset of X and $f(X - U) = f(F) = Y - f(U)$ which implies $f(F) \neq \phi$. Therefore by (2) there is a $\omega\alpha$-closed subset D of Y such that $D \neq Y$ and $f(F) \subset D$. Put $V = X - D$, clearly V is $\omega\alpha$-open set and $V \neq \phi$.Further, $V = X - D \subset Y - f(F) = Y - [Y - f(U)] = f(U)$. □

Theorem 4.10. *If* $f : (X, \tau) \to (Y, \sigma)$ *is somewhat* $\omega\alpha$*-open function and* A *be any open subset of* X*. Then*$f/A : (A, \tau/A) \to (Y, \sigma)$ *is also somewhat* $\omega\alpha$*-open function.*

Theorem 4.11. *If* $f : (X, \tau) \to (Y, \sigma)$*be a function such that* f/A *and* f/B *are somewhat* $\omega\alpha$*-open, then* f*is somewhat* $\omega\alpha$*-open function, where* $X = A \cup B$*,* A *and* B *are open subsets of* X*.*

5 Totally $\omega\alpha$ - Continuous Functions

In this section, we introduce a new class of functions called totally $\omega\alpha$- continuous functions and study some of their properties.

Definition 5.1. *A function* $f : (X, \tau) \to (Y, \sigma)$ *is said to be totally* $\omega\alpha$*- continuous, if the inverse image of every open subset of* Y *is an* $\omega\alpha$*-clopen subset of* X*.*

Example 5.2. *Let* $X = Y = \{a, b, c\}$*,* $\tau = \{X, \phi, \{a\}\}$ *and* $\sigma = \{Y, \phi, \{a\}, \{b, c\}\}$*. Define a function* $f : (X, \tau) \to (Y, \sigma)$ *by*$f(a) = b$*,* $f(b) = a$ *and* $f(c) = c$*.Then* f *is totally* $\omega\alpha$*- continuous function*

Theorem 5.3. *Every perfectly* $\omega\alpha$*- continuous map is totally* $\omega\alpha$*- continuous but converse need not be true in general.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be a perfectly $\omega\alpha$- continuous. Let U be an open set in Y. Then U is $\omega\alpha$-open in Y. Since f is a perfectly $\omega\alpha$- continuous, $f^{-1}(U)$ is clopen in X, implies that $f^{-1}(U)$ is $\omega\alpha$-clopen in X. □

Example 5.4. *In Example 5.2, f is totally $\omega\alpha$- continuous but not perfectly $\omega\alpha$- continuous.*

Theorem 5.5. *Every totally $\omega\alpha$- continuous function is $\omega\alpha$- continuous but converse need not be true in general.*

Example 5.6. *Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a, c\}\}$. Then the identity function $f : (X, \tau) \to (Y, \sigma)$ is $\omega\alpha$- continuous function but not totally $\omega\alpha$- continuous function.*

Remark 5.7. *It is clear that the totally $\omega\alpha$- continuous function is stronger than $\omega\alpha$- continuous and weaker than perfectly $\omega\alpha$- continuous.*

Theorem 5.8. *If $f : (X, \tau) \to (Y, \sigma)$ is totally $\omega\alpha$- continuous function from an $\omega\alpha$-connected space X in to Y, then Y is an indiscrete space.*

Proof. Suppose that Y is not indiscrete space. Let A be a proper non-empty open subset of Y. Then $f^{-1}(A)$ is a non-empty proper $\omega\alpha$- clopen subset of X which is contradiction to the fact that X is $\omega\alpha$-connected. □

Definition 5.9. *A topological space X is said to be $\omega\alpha_2$-space [7], if for every pair of distinct points x and y in X, there exists $\omega\alpha$-open sets M and N such that $x \in N$, $y \in M$ and $M \cap N = \phi$.*

Theorem 5.10. *Let $f : (X, \tau) \to (Y, \sigma)$ be totally $\omega\alpha$- continuous injection map. If Y is T_0, then X is $\omega\alpha_2$-space.*

Proof. Let x and y be any pair of distinct points of X. Then $f(x) \neq f(y)$. Then there exists an open set U containing $f(x)$ but not$f(y)$. Since Y is T_0. Then $x \notin f^{-1}(U)$ and $y \notin f^{-1}(U)$. Since f is totally $\omega\alpha$- continuous,$f^{-1}(U)$ is an $\omega\alpha$-clopen subset of X. Also $x \in f^{-1}(U)$ and $y \in (f^{-1}(U))^c$. Hence X is $\omega\alpha_2$-space. □

Theorem 5.11. *A topological space X is $\omega\alpha$ -connected if and only if every totally $\omega\alpha$- continuous function from a space X in to any T_0-space Y is a constant function.*

Theorem 5.12. *Let $f : (X, \tau) \to (Y, \sigma)$ is totally $\omega\alpha$- continuous and Y be a T_1-space. If A is an $\omega\alpha$-connected subset of X, then $f(A)$ is a single point.*

Theorem 5.13. *A function $f : (X, \tau) \to (Y, \sigma)$ is totally $\omega\alpha$- continuous at a point $x \in X$ if for each open subset V in Y containing $f(x)$, there exists a $\omega\alpha$-clopen subset U in X containing x such that $f(U) \subset V$.*

Proof. Let V be an open subset of Y and let $x \in f^{-1}(V)$. Since $f(x) \in V$, there exists a $\omega\alpha$-clopen set U_x in X containing x such that $U_x \in f^{-1}(V)$. We obtain $f^{-1}(V) = U_{x\in f^{-1}(V)}$. Since arbitrary union of $\omega\alpha$-open sets is $\omega\alpha$-open, $f^{-1}(V)$ is $\omega\alpha$-clopen in X. □

Definition 5.14. *Let X be a topological space. Then the set of all points y in X such that x and y cannot be separated by a $\omega\alpha$-separation of X is said to be the quasi $\omega\alpha$-component of X.*

Theorem 5.15. *Let $f : (X, \tau) \to (Y, \sigma)$ is totally $\omega\alpha$- continuous map from a topological space X in to a T_1-space Y, then f is constant on each quasi $\omega\alpha$-component of X.*

Proof. Let x and y be two points of X that lie in the some quasi $\omega\alpha$-component of X. Assume that $f(x) = \alpha \neq \beta = f(y)$. Since Y is T_1, α is closed in Y and so α^c is an open subset in Y. Since f is totally $\omega\alpha$- continuous,$f^{-1}(\alpha)$ and $f^{-1}(\alpha^c)$ are disjoint $\omega\alpha$-clopen subsets of X. Further $x \in f^{-1}(\alpha)$ and $y \in f^{-1}(\alpha)^c$, which is a contradiction in view of the fact that y must belong to every $\omega\alpha$-clopen set containing x. □

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