FAO Penman-Monteith equation for reference evapotranspiration from missing data

La ecuación de FAO Penman-Monteith para la evapotranspiración de referencia usando datos faltantes

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ABSTRACT

The determination of the amount of water needed for crops is one of the main parameters for correct irrigation planning. In this context the FAO Penman-Monteith method (FAO-PM) has been recommended as the best for the reference evapotranspiration (ETo) estimates. However it is not always possible to have all the necessary data for its application; in this case, alternative criteria must be used for this estimation. In this study, the objective was to estimate ETo by the FAO-PM method with only the maximum and minimum temperature values, here called Simplified FAO-PM, and compare it to the standard method (FAO-PM) with all input data, for Lavras, Minas Gerais State, Brazil. It was observed that the alternative method has the tendency to overestimate the standard method, however, this approach is feasible to estimate ETo for irrigation scheduling in localities where not all input data required for FAO-PM is available.

Key words: evapotranspiration estimate, minimum temperature, maximum temperature, irrigation management.

RESUMEN

La cuantificación de las necesidades de agua del cultivo es un parámetro clave para la planificación del riego. En el presente, la ecuación de Penman-Monteith (FAO-PM) ha sido recomendada como la mejor para estimar la evapotranspiración de referencia (ETo). Sin embargo, no siempre es posible tener todos los datos necesarios para su aplicación, pudiendo en este caso adoptar criterios alternativos para el uso de la misma. Por lo tanto, el objetivo del estudio consistió en estimar ETo por la ecuación FAO-PM con solo los valores de las temperaturas máximas y mínimas, cambiando su nombre por la FAO PM Simplificado, y compararlo con el método estándar (FAO-PM) y con esta ecuación tenía todos los datos de entrada para Lavras, Minas Gerais, Brasil. Se observó que el método alternativo tiende a sobrestimar el método estándar, sin embargo, con la propuesta alternativa es posible estimar ETo para el riego en los lugares donde no siempre tienen la disponibilidad de todos los datos de entrada necesarios para la ecuación FAO-PM.

Palabras clave: estimación de evapotranspiración, temperatura mínima, temperatura máxima, manejo del riego.

Introduction

The development of methodologies to estimate accurately the water need to obtain optimum crop production has become absolutely necessary. For this, the crucial point is the correct quantification of the crop evapotranspiration (ETc). For its quantification it is necessary to know the reference evapotranspiration (ETo) previously; with this information, the ETo is adjusted by the crop coefficient according to the phenological phase, determining the ETc.

The ETo was defined by Doorenbos & Pruitt (1977) as that which occurs in a large ground area completely covered with grass (0.08 to 0.15 m) under active growth, without water restriction, that can be measured in field or estimated by

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mathematical models. Pereira et al. (1997) defined evapotranspiration as a fundamental climatological element which corresponds to the opposite process of rain.

In May, 1990 the Food and Agriculture Organization (FAO) of the United Nations held a meeting of specialists to review its document “Irrigation and Drainage Paper 24” (Doorenbos & Pruitt, 1977) to evaluate new procedures to estimate ET. This panel unanimously recommended the Penman-Monteith model as a new standard for the estimation of ETo, (hereafter FAO-PM). The FAO-PM defines ETo as “the rate of evapotranspiration from a hypothetical crop with an assumed crop height (0.12 m) and a fixed canopy resistance (70 s m$^{-1}$) and albedo (0.23) which would closely resemble evapotranspiration from an extensive surface of green grass cover of uniform height, actively growing, completely shedding the ground and not lacking water” (Pereira et al., 1997).

The need for an accurate and standard method to estimate ETo to predict crop water requirements has been stated by several authors (Martínez-Cob and Tejero-Juste, 2004). A large number of equations for estimating ETo are reported in the literature (Gavilán et al., 2006; Alexandris et al., 2005; DehghaniSanij et al., 2004), but the international scientific community has accepted the FAO-PM equation as the most precise for good results compared to other equations in various regions throughout the world (Garcia et al., 2004; Gavilán et al., 2006). Subsequent papers have demonstrated the superiority of the FAO-PM equation over other methods (Allen et al., 1998) compared to lysimeter measurements, especially for daily calculations (Cai et al., 2007; López-Urrea et al., 2006; Garcia et al., 2004).

For daily ETo calculation, the FAO-PM method requires daily data on maximum and minimum air temperature (Tmax and Tmin), relative humidity (RH), solar radiation (Rs) and wind speed (u). Unfortunately, for many locations such meteorological variables are often incomplete and/or not available. Allen et al. (1998) proposed the use of the Hargreaves (HG) equation as an alternative ETo estimation equation when only air temperature data is available at weather stations. This method behaves best for weekly or longer predictions, although some accurate ETo daily estimations have been reported in the literature (Hargreaves & Allen, 2003).

Barros et al. (2009) evaluated the estimate of ETo for the region of Seropédica, RJ, correlating measurements obtained with weighing lysimeter and FAO-PM, Hargreaves-Samani (HS), Camargo (CA), Priestley-Taylor (PT), Makkink (MA) and Pan Class A (PCA) methods. The values of ETo estimated by FAO-PM, PT and MA were well correlated with those determined by the weighing lysimeter, whereas the HS and CA methods showed unsatisfactory adjustment with the lysimeter data. The errors estimated from the adjusted methods for the region of Seropédica are acceptable for use in design and management irrigation systems.

Reis et al. (2007) compared the estimate of the evapotranspiration in three localities of Espírito Santo State, Brazil during the dry period. For the weather conditions where the work was done, according to the dry and rainy periods, the best methods were: Penman 48 Original, Priestley-Taylor (PT), FAO24 Penman Modified, FAO24 Blaney-Criddle, Turc (61), FAO24 Radiation and Makkink (Mk).

Trajkovic & Kolakovic (2009) evaluated five reference evapotranspiration equations using data from seven humid locations. The equations evaluated include Hargreaves, Thornthwaite, Turc, Priestley-Taylor and Jensen-Haise, evaluated against the corresponding values estimated using the FAO-PM equation. The Turc equation had the lowest weighted RMSD and ranked first; other equations ranked in decreasing order were: Priestley-Taylor, Jensen-Haise, Thornthwaite and Hargreaves. The results obtained from this study indicate very clearly that the Turc equation is most the suitable for estimating reference evapotranspiration at humid locations when weather data are insufficient to apply the FAO-PM equation.

Similar to the present work, Jabloun & Sahli (2008) verified that the FAO-PM model using only maximum and minimum temperatures in general provided satisfactorily good ETo estimates compared to FAO-PM in various locations in Tunisia. Considering all locations, the $R^2$ values were greater than 0.9 and the slope values ranged from 0.96 to 1.06, showing strong relation between ETo-Tmax, Tmin and FAO-PM.

Innumerable other studied methods are present in the literature. For example, Mendonça et al. (2003) verified that the FAO-PM method was the best compared to lysimeter data. Other researchers (Villa Nova et al. (2006), Utset et al. (2004), Droogers & Allen (2002)) have verified that the FAO-PM
method is the most accurate to calculate the ETo, and studied comparisons between FAO-PM and other methods. In studies such as those of Andrade Júnior et al. (2003), Fietz et al. (2005) and Conceição and Mandelli (2005), developed in different regions of Brazil, several methods were evaluated to estimate ETo. The conclusions vary widely among the studies, due to the climatic conditions, leaving it up to the user as to which method to adopt.

According to the above information, it is therefore important to assess the accuracy of the procedures to estimate ETo from missing data. Thus, like the method presented by Allen et al. (1998), this study was carried out to estimate ETo by the FAO-PM method using just maximum and minimum temperature (hereafter Tmax and Tmin, respectively) data, and compare it to the standard method (FAO-PM) having complete data input, in Lavras, Minas Gerais State, Brazil.

Materials and Methods

The data used, which is from 1990 to 2005, was provided by the Principal Climatological Station of Lavras, Minas Gerais State, Brazil (Agreement between Federal University of Lavras-UFLA and National Institute of Meteorology-INMET); its coordinates are latitude: 21°14’ S; longitude: 45°00’ W and altitude: 918.841 m. The region presents a Cwa climate type, according to the Köppen classification, with average annual temperature of 19.4 °C, total annual rainfall of 1,529.7 mm and average annual relative humidity of 76.2% (Brasil, 1992 and Dantas et al., 2007).

The ETo was calculated with all necessary data according to FAO-PM, month to month for every year of the series, as well as using only the data from Tmax (maximum temperature) and Tmin (minimum temperature) of the air (hereafter Simplified FAO-PM); finally it was analyzed by regression. According to Pereira et al. (1997) and Allen et al. (1998), the FAO-PM equation can be calculated from

\[
ETo = \frac{0.408 \Delta (Rn-G)+\gamma \frac{900}{(T+273)} u_2 (es-ea)}{\Delta+\gamma (1+0.34u_2)} \quad (1)
\]

where each of the parameters of this equation is calculated by specific equations described by the same authors cited above.

The Simplified FAO-PM, also presented in the FAO-56 bulletin (Allen et al, 1998) follows the same equation (1), however the parameters for this equation are obtained only on the basis of available data for Tmax and Tmin, besides the geographic coordinates (latitude, longitude) and altitude, whose procedures are described below.

1. Wind Speed (\(u_2\))

As ETo is little sensitive to wind speed, it is safe to assume \(u_2\) as 2 m s\(^{-1}\) as suggested by Allen et al. (1998) or extract from the Climatological Normal for the day of the respective month (Brasil, 1992).

2. Slope of saturation vapor pressure curve (\(\Delta\))

\[
\Delta = \frac{4098\left(0.6108\exp\left(\frac{17.27T}{T+237.3}\right)\right)}{(T+237.3)^2} \quad (kPa \ {°C}^{-1})
\]

where T is the daily average temperature between Tmax and Tmin.

3. Psychrometric constant (\(g\)) obtained by the following equation

\[
\gamma = 0.665 \times 10^{-3} P \quad (kPa \ {°C}^{-1}) \quad (3)
\]

where P is the atmospheric pressure obtained as a function of altitude using

\[
P = 101.3 \left(\frac{293-0.0065z}{293}\right)^{0.26} \quad (kPa) \quad (4)
\]

where z is the local altitude (m).

4. Actual vapor pressure (\(ea\))

This estimate can be obtained assuming that the dewpoint temperature (\(T_{dew}\)) is near the daily Tmin. This statement implies that at sunrise, when the air temperature is close to Tmin, the air is nearly saturated with water vapor and the relative humidity is nearly 100%. Then it is estimated according to the following equation, derived from Tetens’ equation, based on the concept that if the parameter \(es\) is calculated by \(T_{dew}\) its return the value of \(ea\):
ea = 0.6108 \times 10^{\left(\frac{7.5 T_{\text{min}}}{(237.3+7 T_{\text{min}})}\right)} \text{ (kPa)} \quad (5)

5. Solar Radiation (Rs)
This is calculated by Hargreaves’ equation

$$Rs = k_{Rs} \sqrt{T_{\text{max}}-T_{\text{min}}} \cdot Ra \quad \text{(MJ m}^{-2} \text{d}^{-1}) \quad (6)$$

where Ra is the extraterrestrial radiation, whose estimate follows the same procedures presented by Allen et al. (1998) and Pereira et al. (1997) to obtain the net radiation (Rn) for equation 1. The coefficient $k_{Rs}$ is an empirical adjustment coefficient that differs for ‘interior’ or ‘coastal’ regions. For ‘interior’ locations where land masses dominate and air masses are not strongly influenced by a large water body, oceans for example, $k_{Rs}$ assumes a value close to 0.16, while for ‘coastal’ locations, situated on or adjacent to the coast of a large land mass and where air masses are influenced by a nearby water body such as an ocean, $k_{Rs}$ assumes a value close to 0.19. Therefore the previous equation assumes the following formulation in this study

$$Rs = 0.16 \sqrt{T_{\text{max}}-T_{\text{min}}} \cdot Ra \quad \text{(MJ m}^{-2} \text{d}^{-1}) \quad (7)$$

In the same way that was discussed for Ra, the net shortwave radiation (Rns), one component of the net radiation (Rn), does not have modifications for its estimate for equation 1.

6. Clear-sky shortwave radiation (Rso). This parameter is used to calculate the net long wave radiation

$$Rso = \left(0.75 + 2 \times 10^{-5} \cdot z\right) Ra \quad \text{(MJ m}^{-2} \text{d}^{-1}) \quad (8)$$

in which $z$ is altitude (m)

7. Net long wave radiation (Rb)

$$Rb = \sigma \left[\frac{T_{\text{max}}+T_{\text{min}}}{2}\right] \cdot (0.34 - 0.14 \sqrt{ea}) \left(1.35 \frac{Rs}{Rso} - 0.35\right) \quad \text{(MJ m}^{-2} \text{d}^{-1}) \quad (9)$$

where $\sigma$ is the Stefan-Boltzmann coefficient and $T_{\text{max}}$ and $T_{\text{min}}$ are maximum and minimum temperatures on the absolute scale (K).

The performance of the model was evaluated by the determination coefficient of the regression “$r^2$”, the agreement index “$d$” proposed by Willmott et al. (1985) and the confidence index “$c$” proposed by Camargo (Camargo & Sentelhas, 1997).

The index of Willmott et al. (1985), equation 10, which varies between 0 and 1, indicates how many values predicted by the models are equal. An index value closer to 1 (one) indicates better performance, i.e., that the results are similar. The determination coefficient “$r^2$” indicates the model accuracy, i.e., how much of the variation in the dependent variable is explained by the variation of the independent variables.

$$d = 1 - \frac{\sum_{i=1}^{n} (Ye_i - Yo_i)^2}{\sum_{i=1}^{n} (|Ye_i - \overline{Yd}| + |Yo_i - \overline{Yd}|)^2} \quad (10)$$

where $d$ is the agreement index; $Ye_i$ is the ith predicted or estimated value; $Yo_i$ is the ith observed value and $\overline{Yo}$ is the mean of observed values.

The confidence index “$c$” is the product of the coefficient correlation “$r$”, the square root of the determination coefficient, and the agreement index “$d$”, equation 11. The interpretation of the performance by index “$c$” is presented in Table 1.

$$c = r \cdot d \quad (11)$$

<table>
<thead>
<tr>
<th>Confidence index “$c$”</th>
<th>Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&gt; 0.85$</td>
<td>Best - B</td>
</tr>
<tr>
<td>0.76 to 0.85</td>
<td>Very good - VG</td>
</tr>
<tr>
<td>0.66 to 0.75</td>
<td>Good - G</td>
</tr>
<tr>
<td>0.61 to 0.65</td>
<td>Fair - F</td>
</tr>
<tr>
<td>0.51 to 0.60</td>
<td>Bad - B</td>
</tr>
<tr>
<td>0.41 to 0.50</td>
<td>Very bad - VB</td>
</tr>
<tr>
<td>$\leq 0.40$</td>
<td>Worst - W</td>
</tr>
</tbody>
</table>

Table 1. Criterion for interpretation of the confidence index “$c$”.

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Results and Discussion

The comparison between the FAO-PM equation for ETo which was estimated with all necessary data, called in the graph ETo-FAO-PM, and the alternative reference evapotranspiration method using just Tmax and Tmin, called ETo-Simplified FAO-PM in the graph, is depicted in Figure 1 with three proposed statistical parameters and the adjusted regression equation of the trend line. The graphs (Figure 1) contain all data of the series used in this study. Figure 1a presents the linear adjustment and Figure 1b an exponential adjustment, whose best correlation was obtained by the exponential fit $R^2 = 0.8242$. The graphs show the tendency of Simplified FAO-PM to overestimate the values of ETo compared to FAO-PM. Similarly, analyses were also conducted monthly for three month periods in the rainy (October to March) and dry (April to September) season. The results are presented in Table 2. Additionally, the performances of the comparisons in three-month periods are presented graphically in Figures 2 to 5. In a monthly analysis, the best results were observed for the months from August to March, where $R^2$ ranged between 0.6826 and 0.8198 (Table 2); for the other months that correspond to middle of the year the adjustment coefficients were lower ($R^2$ range between 0.5749

![Figure 1](image1.png)

Figure 1. Comparison of the Standard FAO Penman-Monteith reference evapotranspiration (ETo - FAO-PM) and an alternative method using maximum and minimum temperature only (ETo - Simplified FAO-PM) in Lavras, Minas Gerais State, Brazil.

![Figure 2](image2.png)

Figure 2. Comparison of the Standard FAO Penman-Monteith reference evapotranspiration (ETo - FAO-PM) and an alternative method using maximum and minimum temperature only (ETo - Simplified FAO-PM) considering a three month period (January to March) in Lavras, Minas Gerais State, Brazil.
Figure 3. Comparison of the Standard FAO Penman-Monteith reference evapotranspiration (ETo - FAO-PM) and an alternative method using maximum and minimum temperature only (ETo - Simplified FAO-PM) considering the three month period April to June in Lavras, Minas Gerais State, Brazil.

Figure 4. Comparison of the Standard FAO Penman-Monteith reference evapotranspiration (ETo - FAO-PM) and an alternative method using maximum and minimum temperature only (ETo - Simplified FAO-PM) considering the three month period July to September in Lavras, Minas Gerais State, Brazil.

Figure 5. Comparison of the Standard FAO Penman-Monteith reference evapotranspiration (ETo - FAO-PM) and an alternative method using maximum and minimum temperature only (ETo - Simplified FAO-PM) considering the three month period October to December in Lavras, Minas Gerais State, Brazil.
and 0.6704). This suggests that the alternative criterion proposed for the FAO-PM model with $T_{\text{max}}$ and $T_{\text{min}}$ only (Simplified FAO-PM) has better validity for the months from August to March when applied using a monthly scale. By contrast, over a three month period the analysis indicated that the period July–September was better adjusted with linear regression ($R^2 = 0.8415$), Table 2 and Figure 4. However the results presented here were not as good as those presented by Jabloun & Sahli (2008) who considered several locations in Tunisia; their $R^2$ values were greater than 0.9, showing strong correlation between $E_{\text{To}}$ with $T_{\text{max}}$ and $T_{\text{min}}$ only and $E_{\text{To}}$-Standard FAO-PM.

Observing the Figures, in general and as discussed by Villa Nova et al. (2006), the coefficients of determination and correlation indicate the degree of accuracy but do not reveal the precision of the model; it is possible to verify by means of the Figures 1 to 5 and Table 2 that both accuracy, given by the trend line, and precision demonstrated by the dispersion of the data around the fitted 1:1 line or Willmott index, were reasonable in this study. Otherwise, the confidence index shows that the models are very good and good for general analysis and for three-month periods, which are more recommended for its application.

Conceição & Mandelli (2005), studying $E_{\text{To}}$ values estimated with the empirical equations and compared to the FAO-PM method, concluded that the methods employing only air temperature, such as Hargreaves-Samani and Thornthwaite, showed poorer results than the others for the city of Bento Gonçalves, RS, Brazil. This result was also observed by Trajkovic & Kolakovic (2009), when they compared five $E_{\text{To}}$ equations against FAO-PM using data from seven humid locations in Croatia and Serbia; the equations ranked in decreasing order were: Turc, Priestley–Taylor, Jensen–Haise, Thornthwaite, and Hargreaves.

Reis et al. (2007) also found the worst performance of the Hargreaves-Samani method compared to the FAO-PM on a daily scale; therefore, there is a restriction of its use for the weather conditions in the three localities of Espírito Santo State, Brazil during the dry period. However,
considering the results obtained on a weekly scale, all methods studied showed performance ranging from very good to excellent, so there are no restrictions of their use in estimating the ETo for climatic conditions of the study site.

As discussed by Barros et al. (2009) comparing ETo estimates, besides the application of these being limited to the climatic conditions under which they were developed, the estimates were more reliable only for long periods. This suggests that in the present study, perhaps working on a longer time scale in addition to the daily scale could provide better results.

In view of the results presented here and the conclusions obtained by Droogers & Allen (2002), it is concluded that the FAO-PM is a recommended methodology if accurate weather data collection can be expected, but otherwise the ETo-Simplified FAO-PM should be considered. The best FAO-PM of ETo estimates are obtained as a function of their physical-mathematical formulation of the evapotranspiration process and by the greater number of variables considered, which increases the accuracy of the estimates (Allen et al., 1998). But its practical use has been restricted by the requirement for a greater number of climatic elements.

The results imply that for situations where accuracy in weather measurements is expected to be low, it may be better to opt for using a limited data set, in this case only maximum and minimum temperature, than to attempt to establish a full data set. With the reduced data set one can apply the ETo-Simplified FAO-PM equation to simulate ETo, mainly where surface networks are scarce and provide reduced meteorological observations.

**Conclusions**

The alternative method ETo-Simplified FAO-PM has a tendency to overestimate the standard method FAO-PM (with complete data entry) and this application has greater validity for three month periods. However, the ETo-Simplified FAO-PM approach is a feasible alternative to estimate reference evapotranspiration for irrigation scheduling in localities where not all the input data required for FAO-PM is available.

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**Literature Cited**


