Ingeniare. Revista chilena de ingeniería, vol. 16 Nº 3, 2008, pp. 421-427

A GRASP METAHEURISTIC FOR THE ORDERED CUTTING STOCK PROBLEM

UN METAHEURÍSTICO GRASP PARA EL PROBLEMA DE STOCK DE CORTE ORDENADO

Rodrigo Rabello Golfeto[1] **Antônio Carlos Moretti** [2] **Luiz Leduíno de Salles Neto**[3]

Recibido 19 de junio de 2008, aceptado 5 de diciembre de 2008
Received: June 19, 2008 Accepted: December 5, 2008

RESUMEN

Este estudio presenta un nuevo modelo matemático y un procedimiento metaheurístico de búsqueda voraz adaptativa y aleatoria (GRASP, por sus siglas en inglés) para resolver el problema de stock de corte ordenado. Este problema ha sido introducido recientemente en la literatura. Es apropiado minimizar la materia prima usada por las industrias que manipulan inventarios reducidos de productos, tales como las industrias que usan la base justo a tiempo para su producción. En tales casos, los modelos clásicos para resolver el problema de stock de corte ordenado son inútiles. Los resultados obtenidos, mediante experimentos computacionales para un conjunto de ejemplos aleatorios, demuestran que el método propuesto puede ser aplicado a industrias grandes que procesan cortes en sus líneas de producción y no mantienen en stock sus productos.

Palabras clave: Problemas de stock de corte, GRASP, Just-in-time.

ABSTRACT

This study presents a new mathematical model and a Greedy Randomized Adaptive Search Procedure (GRASP) metaheuristic to solve the ordered cutting stock problem. The ordered cutting stock problem was recently introduced in literature. It is appropriate to minimize the raw material used by industries that deal with reduced product inventories, such as industries that use the just-in-time basis for their production. In such cases, classic models for solving the cutting stock problem are useless. Results obtained from computational experiments for a set of random instances demonstrate that the proposed method can be applied to large industries that process cuts on their production lines and do not stock their products.

Keywords: Cutting Stock problem, GRASP, Just-in-time.

INTRODUCTION

The cutting stock problem (CSP) is a classic problem in the area of operations research. It can be defined as the problem of finding the best way of cutting ordered items from stock rolls of width W such that trim loss is minimized and the total demand is satisfied. This is a common problem arising in the production of paper [5, 15], steel [7, 29], windows [26], etc. The cutting stock problem was one of the very first applications of operations research methods. It was first studied by Kantorovich in the thirties. Paull and Walter [24], Metzger [23] and Eilon [7] later dealt with problems of the same nature.

Besides the trim loss, other costs or restrictions can be relevant in an industry that processes cuts on its production line, such as setup costs and the maximum number of open stacks during the cutting process. For instance, an industry can organize the stacks of final products by customer or by the product's specifications – its width, in the case of the one-dimensional cutting stock problem. Several authors have dedicated themselves to developing methods of obtaining adequate solutions to the cutting stock problem with constraints on the number of open stacks [1, 2, 4, 11, 18, 21, 22, 25, 30 and 31].

1 Universidade Federal Fluminense. Departamento de Ciência da Computação. Avenida dos Trabalhadores, 420, Vila Santa Cecília, 27255-250, Volta Redonda. Rio de Janeiro, Brasil. Email: rodrigo.golfeto@gmail.com
2 Universidade Estadual de Campinas. Departamento de Matemática Aplicada. Rua Sergio Buarque de Holanda, 651, Cidade Universitária - Barão Geraldo Caixa Postal: 6065, 13083-859. Campinas, São Paulo, Brasil. E-mail: moretti@ime.unicamp.br
3 Universidade Federal de São Paulo. Departamento de Ciência y Tecnologia. Avenida Mário Covas, 610, 12231-280. São José dos Campos, São Paulo, Brasil. E-mail: luiz.leduino@unifesp.br

Ragsdale and Zobel [25] dealt with the problem as applied to an American manufacturer of wooden windows and doors, but it can be applied to industries that work on a just-in-time or mass-customization basis: only one open stack is permitted beside the cutting machinery. In this case, one stack is opened for every new order. The authors proposed a method based on genetic algorithms. The chromosomes are permutations of clients' orders and of items within each order, and their fitness function measures the amount of waste for a given solution. An additional local-search heuristic (PARTCUT) helps to improve the pattern sequence. In their study, Ragsdale and Zobel [25] named this problem the Ordered Cutting Stock Problem (OCSP).

Alves and Valerio de Carvalho [1] developed three mathematical models and used a branch-and-price approach to a similar problem: items of one order can be cut with items of other orders on at most two stock pieces; there can be no more than two orders partially cut from the same stock piece; a pattern composed of items of more than one order can be used at most once; orders partially cut on a stock piece must be at the head or tail of that stock piece. Alves and Valerio de Carvalho [1] also considered that several objects of different sizes were in stock.

In this study, we present a GRASP metaheuristic that minimizes the number of processed objects in an Ordered Cutting Stock Problem. GRASP is a multi-start metaheuristic that uses a random greedy heuristic and a local-search procedure for each repetition. Aiming at obtaining the optimal solution to the OCSP, we coupled the GRASP with a path-relinking procedure.

The outline of this research paper is as follows: Section "The Ordered Cutting Stock Problem" briefly introduces the ordered cutting stock problem. Section "Greedy Randomized Adaptive Search Procedure" presents a general introduction to the grasp metaheuristic. Section "The Grasp Solution to the OCSP" is devoted to describing our approach: a path-relinking grasp for the OCSP. Experimental results are presented and analyzed in Section "Computational Experiments". Finally, in section "Conclusions and Perspectives", we conclude the paper with a summary of the study and an overview of possible future research.

THE ORDERED CUTTING STOCK PROBLEM

The Standard Cutting Stock Problem is characterized by cutting stock rolls of width W (called objects) into smaller rolls of width W_i (where $W>W_i$), aiming at satisfying the demand d_i for each one of these m items. According to Dyckhoff's typology [6], this problem is classified as 1/V/I/R. Each combination of items in an object is called the cutting pattern, and each change in the cutting pattern has a setup cost to prepare the cutting machine. The mathematical model for minimizing trim-loss costs can be stated as follows:

$$\text{Minimize} \quad c_1\left(W \times \sum_{j=1}^{n} x_j - \sum_{i=1}^{m} w_i d_i\right)$$

$$\text{s.t.:} \quad \sum_{j=1}^{n} a_{ij} x_j \geq d_i, \qquad i = 1,\dots,m.$$

$$x_j \in N, \qquad j = 1,\dots,n.$$

where c_1 is the cost of the trim loss; a_{ij} is the number of times item i appears in the jth cutting pattern; x_j is the number of objects processed with cutting pattern j.

However, as observed in the introduction, this model is inappropriate for industries that work with reduced inventories. Let us consider the following example. An industry produces four types of products with the following widths: $W_1 = 2$; $W_2 = 4$; $W_3 = 5$; and $W_4 = 7$. We shall presume that a large number of the objects in stock have a width of W = 20 and that this industry possesses the four-client portfolio that follows in table 1:

Table 1. Clients' Orders.

Orders	d_1	d_2	d_3	d_4
1	2	1	1	1
2	5	4	0	2
3	2	0	2	0
4	4	3	1	1

We need to determine the minimum trim-loss cutting plan by which all the items of a client's order are cut in sequence. Figure 1 presents a solution to this problem, whereby the clients' items are cut in the following sequence: 3,2,1,4 – i.e., Client 3's items are cut first because the remaining object still will have space for cutting two items that were ordered by Client 2. The process continues until all the clients' orders are satisfied.

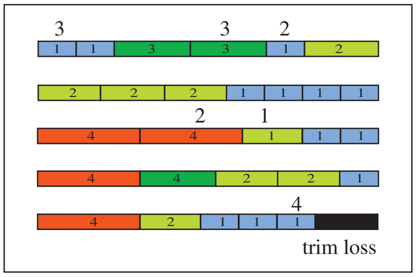

Figure 1. Example of a practical solution.

In more formal terms, we apply the following notation to the Ordered Cutting Stock Problem:

- NC - is the number of clients' orders;
- w_i - is the size of item i;
- d_{ik} - is the amount of items i ordered by Client k;
- X_{jk} - is the integer decision variable that determines the number of processed objects having Client k's cutting pattern j.
- y_{kl} - is the binary integer decision variable that is equal to 1 if one object is cut with the client's items k and l;
- a_{ijk} - is the number of times item i appears in the jth cutting pattern of Client k;
- N_K - is the number of cutting patterns of Client k;
- α_{ijkl} - is the number of items i of Client k using cutting pattern j in the object shared by clients k and l;
- N_{KL} - is the number of cutting patterns using items of clients k and l, k = 1,…..,NC, l = k + 1,...,NC.

With this notation, we propose the following mathematical model for the Ordered Cut¬ting Stock Problem:

$$\text{Minimize} \quad \sum_{k=1}^{NC}\sum_{j=1}^{NK} x_{jk} + \sum_{k=1}^{NC}\sum_{l=k+1}^{NC} z_{kl}$$

$$s.t.: \quad \sum_{j=1}^{NK} a_{ijk} x_{jk} + \sum_{l=1}^{NC}\sum_{j=1}^{NKL} a_{ijkl} z_{lk} = d_{ik} \qquad (1)$$

$$i = 1,\ldots,m, k = 1,\ldots,NC.$$

$$\sum_{k=l+1}^{NC} z_{kl} \le 1 \quad k = 1,\ldots,NC. \qquad (2)$$

$$\sum_{k=1}^{l-1} z_{kl} + \sum_{k=l+1}^{NC} z_{lk} \le 2 \quad l = 1,\ldots,NC. \qquad (3)$$

$$x_{jk} \ge 0, x_{jk} \in Z \quad j = 1,\ldots,NK, k = 1,\ldots,NC. \qquad (4)$$

$$x_{kl} \in \{0,1\} \quad k = 1,\ldots,NC, l = k+1,\ldots.,NC. \qquad (5)$$

Constraint 1 guarantees that the demand will be met. Constraints 2 and 3 guarantee that no more than one cutting pattern will share the same pair of clients.

GREEDY RANDOMIZED ADAPTIVE SEARCH PROCEDURE

The GRASP (Greedy Randomized Adaptive Search Procedure) metaheuristic [9] is a multi-start or iterative process, in which each repetition consists of two phases: construction and local search. The best solution is updated for each repetition. Festa and Resende [10] present a detailed analysis, listing the various applications described in the literature, as well as the main strategies and characteristics of the GRASP. Basically, the GRASP metaheuristic operates as follows:

```
Procedure - GRASP (Max-iterations, Seed)
FOR i=1,..., Max-iterations, DO
Solution – Greedy Randomized Construction (Seed)
Solution – Local Search (Solution)
Update Solution (Solution, Best Solution)
END FOR
RETURN Best Solution
END GRASP
```

A strategy often used in the random greedy phase is the creation of a restricted candidate list (RCL). A greedy function *fg* is used to create the RCL. A list of candidates is elaborated that satisfies a certain rule or parameter that is sought, such as having an *fg* less than or equal to a certain amount. The initial solution for the iteration is randomly chosen from this restricted candidate list. In the next section, we detail our implementation of the RCL for the OCSP.

Laguna and Marti [20] used a path-relinking strategy that was previously incorporated in tabu-search and scatter-search metaheuristics [15] to explore paths relinking solutions obtained in a GRASP metaheuristic. Path-relinking can be applied as a post-optimization step to all pairs of elite solutions or as a strategy to intensify each local optimum obtained after the local-search phase. Resende [26] argues that applying path-relinking as an intensification strategy to each local optimum seems to be more effective than simply using it as a post-optimization step. In this context, path-relinking is applied to pairs (x_1,x_2) of solutions, where x1 is the locally optimal solution that is obtained after a local search and x2 is one of a few elite solutions randomly chosen from a pool with a limited MaxElite number of elite solutions found during the search.

In the section next, we explain how we incorporated the path-relinking strategy to intensify the search for better solutions for each repetition.

THE GRASP SOLUTION TO THE OCSP

For each repetition of our GRASP (hereafter named GRASP-OCSP), we generated a solution using the constructive FFD heuristic to determine – from a greedy heuristic standpoint – which client should be the next in the sequence. More specifically, we considered each client's order as a bin-packing problem with an infinite number of W-size bins and one bin the size of the surplus of the last processed object of the previous order (if the surplus was greater than zero). To choose the next client in the sequence for each repetition, we used the best solution that was found (omin) – i.e., the smallest number of objects to satisfy the order of a certain client – and the worst solution that was found (omax) – i.e., the largest number of objects determined using the FFD heuristic – for a certain client that was not yet in the sequence in order to determine the RCL, according to the pseudo-code below:

```
Procedure - Generate Random Solution
    Randomly SELECT a client to enter the sequence
        WHILE there are clients outside the list, DO
        CALCULATE the cost of each client's entrance into the sequence using FFD
        CALCULATE BRCL = omin + 0.6(omax - omin)
        Randomly SELECT a client with cost < BRCL to enter the sequence
    END WHILE
END Procedure
```

Using the above procedure, we generated 80 solutions for each GRASP-OCSP repetition. Soon thereafter, we conducted a 2opt local search using the current 80 solutions and updating the Elite group, which is composed of the 20 best solutions obtained up to the moment. We then conducted 40 path-relinking tests using the Elite solutions according to the pseudo-code below, where f is the objective function (i.e, the number of processed objects):

```
Procedure - Path-relinking
    Randomly SELECT from the Elite group two solutions S1 and S2 – vectors with NC coordinates
    S*□ S1
    Randomly SELECT one coordinate n (1 < n < NC)
    REORDER vector S1 so that coordinate n is in the first position
    In solution S2, SEARCH for the coordinate m that possesses the same content as coordinate n of S1
    REORDER vector S2 so that coordinate m is in the first position
    FOR i=2,...,NC DO
        S2(i)=S1(i)
        RESTORE solution S2 to its original form
        IF f (S) >f (S2), THEN S*   S1
        REORDER vector S2 so that coordinate m is in the first position
    END FOR
END Procedure
```

After the 40 attempts to find better solutions using path-relinking, we updated the Elite group and conducted a 2opt local search using the new solutions that now composed the Elite group. Reaching the maximum number of iterations, we discovered that the best solution in the Elite group is the solution presented by the OCSP-GRASP.

COMPUTATIONAL EXPERIMENTS

In order to evaluate our approach, random cases for the one-dimensional cutting prob¬lem were generated by CUTGEN1, which was developed by Gau&Wäscher [8]. We generated 18 classes of problems by combining various CUTGEN1 parameters:

- V_1 was assigned the values 0.01 and 0.2;
- V_2 was assigned the values 0.2 and 0.8;
- The number of patterns in the original cutting plan for each client (denoted by m) was set to 10.
- The ICPRB (in this case representing the number of clients NC for each instance) was assigned the values 50, 80 and 100.
- Dbar (in this case, the average order of each client) was assigned the values 20 and 100

Table 2 shows the parameters used for each class.

To check the quality of the GRASP-OCSP, we adopted the following procedure for each case:

- we transformed the ordered cutting stock problem into a classic cutting stock problem (CCSP), using the clients' orders for each item;
- we relaxed the integer restrictions for each variable;
- we used the Gilmore and Gomory strategy [12,13] to obtain the optimal solution x* for the relaxed problem;
- we used the smallest integer greater than or equal to x* as the inferior bound of the CCSP inf_csp = $\lfloor x^* \rfloor$; inf_csp is also the inferior bound of the OCSP.

Table 3 shows (in average percentages for each class) how much the solution found by the GRASP-OCSP method varied from the inferior bound (infcsp). It also shows the average computational time.

Table 2. Randomly Generated Classes and their parameters.

Class	NC	m	Dbar	v_1	v_2
1	50	10	20	0.01	0.20
2	50	10	20	0.01	0.80
3	50	10	20	0.20	0.80
4	50	10	100	0.01	0.20
5	50	10	100	0.01	0.80
6	50	10	100	0.20	0.80
7	80	10	20	0.01	0.20
8	80	10	20	0.01	0.80
9	80	10	20	0.20	0.80
10	80	10	100	0.01	0.20
11	80	10	100	0.01	0.80
12	80	10	100	0.20	0.80
13	100	10	20	0.01	0.20
14	100	10	20	0.01	0.80
15	100	10	20	0.20	0.80
16	100	10	100	0.01	0.20
17	100	10	100	0.01	0.80
18	100	10	100	0.20	0.80

Table 3. Results obtained for each class using the OCSP-GRASP.

Class	Inf_CSP	GRASP	GAP	T(s)
1	4421,9	4441,2	0,44	18,5
2	16734,7	16989,5	1,52	7,3
3	22255,5	22486,9	1,04	5,4
4	22856	22960	0,46	164,3
5	89575,2	91562,1	2,22	112,8
6	114222,5	115633,9	1,24	104,9
7	4452,5	4470,2	0,4	20
8	17283,1	17602,6	1,85	8,4
9	22896,5	23246,4	1,53	6,7
10	22902,9	23003,7	0,44	152,6
11	88623,7	90082,7	1,65	127,7
12	112262	113924	1,48	106,4
13	8827	8870	0,49	89,6
14	36142,2	36628,7	1,35	50,4
15	45258,7	45784,8	1,16	44,6
16	43901,8	44077,5	0,4	430,1
17	190994,8	193378,8	1,25	247
18	213818,3	217203,6	1,58	238,2

The test cases generated, and results obtained, for each problem are available at www.otimizacao.net.

CONCLUSIONS AND PERSPECTIVES

In this study, we presented a new mathematical model and GRASP metaheuristic for the Ordered Cutting Stock Problem. The results obtained are very close to the bottom limits of the Classic Cutting Stock Problem, which shows that the proposed method generates high-quality solutions, even for problems with a large number of clients. While we worked with cases involving 50, 80 and 100 clients, Ragsdale and Zobel [25] tested their method on real problems involving 50 jobs, and Alves and Valério de Carvalho [1] worked with random cases involving up to 30 clients. Therefore, this study's main contribution is that it is the first approach to the ordered cutting stock problem that achieves precise results for cases involving huge amounts of client orders. Finally, we consider the proposed method extremely viable for large industries that process cuts on their production lines on a just-in-time basis.

ACKNOWLEDGEMENTS

The authors have been partially supported by M.E.C. (Spain), Project MTM2007-063432. The second author is also supported by CNPq (Brasil), Project 307907/2007-4.

REFERENCES

[1] C. Alves and J. M. Valério de Carvalho. "New integer programming formulations and an exact algorithm for the ordered cutting stock problem". Journal of the Operational Research Society, pp. 1-12. 2007.

[2] J. Becceneri, H. H. Yanasse and N. Soma. "A method for solving the minimization of the maximum number of open stacks problem within a cutting process". Comput Op. Res. Vol. 31, pp. 2315-2332. 2004.

[3] M. Bellmore and G. L. Nemhauser. "The Traveling Salesman Problem: A Survey". Operations Research. Vol. 16 Nº 3, pp. 538-558. 1968.

[4] G. Belov. "Problems, models and algorithms in one and two-dimensional cutting". PhD thesis. University of Dresden. 2003.

[5] A. Diegel. "Cutting paper in Richards Bay: dynamic local and global optimization in the trim problem". Orion. Vol. 3, pp. 42-55. 1988.

[6] H. Dyckhoff. "A typology of cutting and packing problems". European Journal of Operation Research. Vol. 44, pp. 145-159. 1990.

[7] S. Eilon. "Optimizing the shearing of steel bars". Journal of Mechanical Engineering Science. Vol. 2, pp. 129-142. 1960.

[8] T. Gau and G. Wascher. "CUTGENI: A Problem Generator for the Standard One-dimensional Cutting Stock Problem". European Journal of Operational Research. Vol. 84, pp. 572-579. 1995.

[9] T.A. Feo and M.G.C. Resende. "A probabilistic heuristic for a computationally difficult set covering problem". Operations Research Letters. Vol. 8, pp. 67-71. 1989.

[10] P. Festa and M.G.C. "Resende "GRASP: An annotated bibliography". In C.C. Ribeiro and P. Hansen, editors, Essays and Surveys in Metaheuristics, pp. 325-367. 2002.

[11] A. Fink and S. Voß "Applications of modern heuristic search methods to pattern sequencing problems". Comput. Op. Res. Vol. 26, pp. 17-34. 1999.

[12] P.C. Gilmore and R.E. Gomory. "A Linear Programming Approach to the Cutting Stock Problem". Operations Research. Vol. 9, pp. 849-859. 1961.

[13] Gilmore and R.E. Gomory. "A Linear Programming Approach to the Cutting Stock Problem". Operations Research. Vol. 11, pp. 863-888. 1963.

[14] F. Glover. "Tabu search and adaptive memory programing - Advances, applications and challenges". In R.S. Barr, R.V. Helgason, and J.L. Kennington, editors, Interfaces in Computer Science and Operations Research, pp. 1-75. 1996.

[15] F. Glover, M. Laguna and R. Marti. "Scatter Search and Path Relinking: Advances and Applications Book Series". International Series in Operations Research and Management Science. Vol. 57, pp 1-35. 2000.

[16] R. Haessler. "Controlling Cutting Pattern Changes in One-Dimensional Trim Problems". Operations Research. Vol. 23, pp. 483-493. 1975.

[17] C.J. Hardley. "Optimal cutting of zinc-coated steel strip". Operational Research. Vol. 4, pp. 92-100. 1976.

[18] M. Johnson, C. Rennick and E. Zak. "One-dimensional cutting stock problem in just-in-time environment". Pesquisa Operacional. Vol. 19, pp. 145-158. 1999.

[19] R.E. Johnston. "Rounding algorithms for cutting stock problems". Asia-Pacific Journal of Operational Research. Vol. 3, pp. 166-171. 1986.

[20] M. Laguna and R. Martí. "GRASP and Path Relinking for 2-Layer Straight Line Crossing Minimization". INFORMS Journal on Computing. Vol. 11 Nº 1, pp. 44-52. 1999.

[21] A. Linhares and H.H. Yanasse. "Connections between cutting pattern sequencing, VLSI design, and flexible machines". Comput. Op. Res. Vol. 29, pp. 1759-1772. 2002.

[22] O. Madsen. "An application of travelling-salesman routines to solve pattern-allocation problems in the glass industry". Journal Op. Res. Soc. Vol. 39, pp. 249-256. 1988.

[23] R.W. Metzger. "Stock Slitting". Elementary Mathematical Programming. Wiley. 1958.

[24] A.E. Paull and J.R. Walter. "The trim problem: an application of linear programming to the manufacture of news-print paper". Presented at Annual Meeting of Econometric Society, Montreal, pp. 10-13. 1954.

[25] C. Ragsdale and C. Zobel. "Ordered Cutting Stock Problem". Decision Sciences. Vol. 35, pp. 83-100. 2004.

[26] M.G.C. Resende. "An Introduction to GRASP". XXXIX Simpósio Brasileiro de Pesquisa Operacional. 2007.

[27] H. Stadler. "A one-dimensional cutting stock problem in the aluminum industry and its solution". European Journal of Operational Research. Vol. 44, pp. 209-223. 1990.

[28] S. Umetani, M. Yagiura and T. Ibaraki. "One Dimensional Cutting Stock Problem to Minimize the Number of Different Patterns", European Journal of Operational Research. Vol. 146 Nº 2, pp.388-402. 2003.

[29] G. Wascher, P. Carow and H. Muller. "Entwicklung eines flexiben Verfahrens für Zuschneideprobleme in einem Kaltwalzwerk". Zeitschrif für Operations Research. Vol. 29. B209-B230. 1985.

[30] H.H. Yanasse. "On a pattern sequencing problem to minimize the maximum number of open stacks". European Journal of Operational Research. Vol. 100, pp. 454-463. 1997.

[31] H.H. Yanasse and M. Lamosa. "An integrated cutting stock and sequencing problem". European Journal of Operational Research. Vol. 183, pp. 1352-1370. 2007.