On the importance of nonlinear systems in science and engineering

Multiple natural phenomena are out of equilibrium, such as the chemical reactions, the dynamic ocean-earth-atmosphere, or like in any living system, just to mention a few. In these systems there is competition between the injection and dissipation of any measurable physical quantity, such as energy or matter. Generally, the way to study these phenomena is based on nonlinear systems. That means, the systems are described by a set of equations in which their variables are non-linearly related. The type of equations depend whether the systems have discrete nature (maps) or continuous nature (differential equations).

In the last decades there has been a great dedication in the scientific community to formalizing and understanding these types of systems, colloquially called complex-systems\textsuperscript{1-3}. In fact, the nonlinear science\textsuperscript{3} has emerged as such that its principles have been successfully applied from biology\textsuperscript{2} to social systems\textsuperscript{2}. This is due to its universal way to understand the systems under study. In addition, one of the keys on the methodology behind this new science is to try to reduce the prototype model to the minimum number of variables that the phenomenon under study is described as simple as possible, but not trivially. Let us remark that the Nobel Prize Richard Feynman who is attributed for the quotation: “The same equations have the same solutions”\textsuperscript{4}, which implies that if one solves a mathematical problem may apply it into different situations. This concept was essential in the study of pattern formation, since it is possible to distinguish the same patterns in different areas of knowledge. For example, dendrite formation can be observed in thunderstorms, or when one breaks a glass with a nail with a hammer, or in super-conductors in the presence of external fields, or just in some European trees. Consequently, one can formulate a system of universal equations that describe this phenomenon, so that the essence of the prototype systems is in the parameters of the equations.

In the case of engineering, nonlinear systems appear naturally. The two classical examples are within the field of fluid dynamics and electronics. In the first case, it has tremendous importance, because it was the precursor of nonlinear science. Indeed, the studies of instabilities began experiencing and understanding in the context of fluid, being the instability of Rayleigh-Benard\textsuperscript{1} and the Taylor-Couette\textsuperscript{1} the most well known. In fact, in the sixties the convection’s problem resulted the first modern study of chaos\textsuperscript{5}. This study was done by Edward Lorenz, and its system of three nonlinear ordinary differential equations generated a new way to understand the world. This study showed that there are not necessary multiple modes to obtain complex states in fluids\textsuperscript{6}; and although if the initial conditions are known deterministic systems are primarily predictable, in practice this predictability can be difficult to obtain due to the sensitivity to those conditions. Moreover, with the development of chaos theory we have learned that there may be an underlying order within disorder, manifesting

\textsuperscript{4} https://es.wikiquote.org/wiki/Richard_Feynman
some times in morphologically attractive shapes like *shrimps*\(^7\) or *mummies*\(^8\). Chaos theory has been applied in countless fields, and thanks to electronic engineering, it has been simply tested in circuits\(^3,9\). A very interesting application from the technological point of view is the encryption of information using chaotic series.

Because today this field of study is transversal and clearly useful, I think it is necessary to include a course on nonlinear systems in the undergraduate curriculum of basic sciences as well as in engineering. At least as an optional course in a first stage. For our students, the basic knowledge of how to understand these systems can be advantageous from the point of view of their academic training as well as for future employments. From the universe of textbook, one that I am really liked for the first course it is the *Strogatz*\(^{10}\). This book is didactic, and the different topics of nonlinear systems are treated in a progressively and comprehensive manner. In addition, an interesting point for students in the study of complex systems is that one can easily perform simulations in the classroom, with unexpected results. The traditional prototype is the logistic map\(^{10}\), which is a quadratic map \[x_{n+1} = r x_n (1 - x_n),\] which can generate steady states, periodic or chaotic ones, depending of the value of the parameter \(r\); and one can explore it with a scientific calculator!

As a final comment of this editorial of the engineering’s journal of the Universidad de Tarapacá, I would like to emphasize again that we are in a time when complex systems are part of daily life, therefore our University should rise to the occasion, and we have to find a way to transmit and create knowledge in this direction for our students so they can be a part of this exciting scientific-technological challenge.

David Laroze, Ph.D.
Instituto de Alta Investigación
Universidad de Tarapacá
Arica, Chile
david.laroze@gmail.com

---