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# ON THE REPRESENTATION TYPE OF CERTAIN TRIVIAL EXTENSIONS

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**Abstract**

*Let $A \cong kQ/I$ be a basic and connected finite dimension algebra over closed field $k$. In this note show that in case $B = A[M]$ is a tame one-point extension of a tame concealed algebra $A$ by an indecomposable module $M$, then the trivial extension $T(B) = B \propto DB$ is tame if and only if the module $M$ is regular.*

## 1. Introduction.

Throughout this paper, $k$ denotes an algebraically closed field. By algebra $A$, we mean always a basic, connected and finite dimensional algebra over $k$ ( associative with 1). We denote by $modA$ the category of finitely generated right $A$-modules, and $\mathcal{D}^b(A)$ the derived category of bounded complexs over the abelian category $modA$ (see [H]).

The concept of repetitive algebra was introduced by Hughes - Waschbush ([HW]) in 1983, where their main interest was to obtain the classification of the finite representation self-injective algebras. In section 2 we recall some known facts about repetitive algebra. In this note, we will use the properties of repetitive categories to study the representation type of the trivial extension $T(B) = B \propto DB$, where $B$ is a one-point extension of a tame concealed algebra by an indecomposable module.

In section 3 we establish our main theorem on the representation type of the trivial extension $T(B)$. For that purpose, we prove that there exist a strong relation between the trivial extension $T(B)$ and the class of clannish algebras introduced by Crawley-Boevey in [C-B]. As a consequence of our main theorem we show that all tree algebra with non-negative Euler form $\chi_A$ of corank $\chi_A \leq 2$, have trivial extension of tame representation type.

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## 2. Preliminaries.

We recall that a quiver $Q = (Q_0, Q_1)$ is an oriented graph, where $Q_0$ is the set of vertices and $Q_1$ is the set of arrows. The ordinary quiver associated to an algebra $A$ will be denote by $Q_A$. The $k$-algebra $A$ will be called triangular when $Q_A$ has no oriented cycles. For each vertex $i$ of $Q_A$, we shall denote by $e_i$ the corresponding primitive idempotent of $A$, and by $S_i$ the corresponding simple $A$-module. We denote $P_i$ (respectively $I_i$) the projective cover (respectively, the injective envelope) of $S_i$. A bound quiver algebra $A \cong kQ/I$ will sometimes be considered as a $k$-category.

Let $\mathcal{H}$ be a Krull-Schmidt category. By definition, the quiver $\Gamma(\mathcal{H})$ of $\mathcal{H}$ has as vertices the isomorphism classes $[M]$ of indecomposable objects $M \in \mathcal{H}$, and there are many arrows $[M] \longrightarrow [N]$ as the dimension of the space of irreducible maps from $M$ to $N$ in $\mathcal{H}$ (see $VII.1$ [ARS]). If $\mathcal{H}$ =mod$A$ or $\mathcal{D}^b(A)$, then $\Gamma(\mathcal{H})$ is a translation quiver (see 2.1 in [R]). The quiver $\Gamma(modA)$, or $\Gamma_A$, is called the Auslander-Reiten quiver of $A$. A translation quiver $\Gamma$ is called a tube (see $VIII.4$ in [ARS]), if it contains cyclic paths and its topological realization is $|\Gamma| = S^1 \times \mathbf{R}_0^+$ (where $S^1$ is the unit circle and $\mathbf{R}_0^+$ is the set of non-negative real numbers). A $k$-category $A$ is called $\widetilde{\mathbf{A}}$-free whenever it contains no full sub category $B \cong kQ$ where the underlying graph of $Q$ is $\widetilde{\mathbf{A_n}}$, for some $n$.

For the basic definitions and results of tilting theory, we refer the reader to [A1]. Two finite-dimensional $k$-algebras $A$ and $B$ are called tilting-cotilting equivalent, if there exist a sequence of algebras $A = A_0, A_1, ..., A_{m+1} = B$ and a sequence of modules $T^i_A$ $(0 \leq i \leq m)$ such that $A_{i+1} = EndT^i_{A_i}$ and $T^i_{A_i}$ is either a tilting or cotilting module (see [A1]).

The one-point extension (respectively, coextension) of an algebra $A$ by an $A$-module $M$ will be denoted by $A[M]$ (respectively, $[M]A$). Let $A$ be a triangular algebra and $i$ a sink in $Q_A$. The reflection $S_i^+ A$ (see [HW]), of $A$ is defined as the quotient of the one-point extension $A[I_i]$ by the bilateral ideal generated by $e_i$. Dually, starting with a source $j$, we define the reflection $S_j^- A$.

By a **polynomial-growth critical algebra**, shortly **pg-critical** algebra (see 3 in [NS]) we mean an alebra $A$ satisfying the following conditions:

1) $A$ or $A^{op}$ is of one of the following form:

$$C[M] = \begin{bmatrix} k & M \\ 0 & C \\ & \end{bmatrix}, \text{ or } \quad C[N,t] = \begin{bmatrix} k & k & \dots & k & k & k & N \\ & k & \dots & k & k & k & 0 \\ & & \ddots & \vdots & \vdots & \vdots & \vdots \\ & & & k & k & k & 0 \\ & & & & k & 0 & 0 \\ 0 & & & & & k & 0 \\ & & & & & & C \\ & & & & & & \end{bmatrix}$$

where $C$ is a representation infinite tilted algebra of type $\widetilde{\mathbf{D_n}}$ with $(4 \leq n)$, with a complete slice in the preinjective component, and $M$ (respectively, $N$) is an indecomposable regular $C$-module of regular length 2 (respectively, regular length 1) lying in a tube $T$ in $\Gamma_C$ having $n-2$ rays, and $t+1$ $(2 \leq t)$ is the number of objects in $C[N,t]$ which are not in $C$.

2) Every proper convex sub category of $A$ is of polynomial growth.

In particular, we say that the algebra $A$ is 2-tubular if $A = \widetilde{\mathbf{D_n}}[M]$, where $M \in ind\widetilde{\mathbf{D_n}}$ is regular indecomposable of length 2 lying in a tube $T$ in $\Gamma_{\widetilde{\mathbf{D_n}}}$ having $n-2$ rays.

**Proposition 2.1** (1.4 **in** [**P1**]). *A pg-critical algebra A is derived-equivalent to an algebra given by the following quiver:*

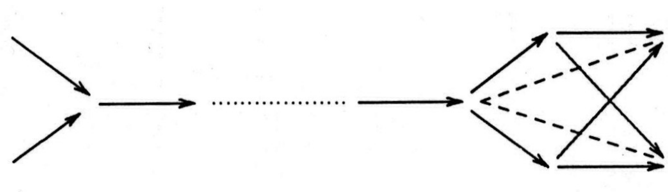

*Figure 1.*

*With the commutative relations, indicated by dotted edges.*

Let $A$ be a finite-dimensional $k$-algebra, and $D = Hom_k(-,k)$ denote the standard duality on $modA$. The *repetitive algebra* $\widehat{\mathbf{A}}$ (see [HW]) of $A$ is the self-injective, locally finite-dimensional algebra without identity, defined by:

$$\widehat{\mathbf{A}} = \begin{pmatrix} \ddots & & & & 0 \\ & A & & & \\ & DA & A & & \\ & & DA & A & \\ 0 & & & & \ddots \end{pmatrix}$$

where matrices have only finitely many non-zero entries, addition is the usual addition of matrices, and multiplication is induced from the canonical bimodule structure of $DA = Hom_k(A, k)$ and the zero map $DA \otimes DA \to 0$.

It was proved in [W] that if $T_A$ is a tilting module and $B = EndT_A$, then $\underline{mod}\widehat{\mathbf{A}} \cong \underline{mod}\widehat{B}$, where $\underline{mod}$ denote a stable category in the sense of chapter $\underline{X}$ in [ARS].

The repetitive algebra $\widehat{\mathbf{A}}$ was introduced as the Galois covering (see [G]) of the trivial extension $T(A) = A \propto DA$ of $A$ by its minimal injective cogenerator $DA$. Let $\nu$ the Nakayama automorphism of $\widehat{\mathbf{A}}$ and $G =< \nu >$. We consider $\widehat{\mathbf{A}}$ as $k$-category, then we have the Galois cover functor: $F : \widehat{\mathbf{A}} \longrightarrow (\widehat{\mathbf{A}}/G)$, where each element of $\widehat{\mathbf{A}}$ corresponds to an orbit. This functor induces the push-down functor $F_\lambda : mod\widehat{\mathbf{A}} \longrightarrow mod(\widehat{\mathbf{A}}/G)$ and pull-up functor $F_. : mod(\widehat{\mathbf{A}}/G) \longrightarrow mod\widehat{\mathbf{A}}$, and by 2.2 in [HW] we know that $T(A) \cong \widehat{\mathbf{A}}/G$.

A $k$-algebra $\widehat{\mathbf{A}}$ is called $(\nu_A)$-exhaustive, when the push-down functor $F_\lambda : mod\widehat{\mathbf{A}} \longrightarrow modT(A)$ associated to the Galois cover functor $F : \widehat{\mathbf{A}} \longrightarrow T(A)$ is dense.

We say that the $k$-algebra $A$ is of locally finite support, if for each indecomposable projective module $P$, the isomorphism class of the indecomposable projective module $P'$ is such that the number of indecomposable module $M$, with $Hom_A(P, M) \neq 0$ and $Hom_A(P', M) \neq 0$ is finite.

In particular, in [LDS] it is show that: If $\widehat{A}$ is of locally finite support if and only if the $gldim\, A$ (global dimension) is **strong** and finite, that is, the complexes of the derived category has bounded length.

In [LDS] it was proved that if a $k$-algebra $A$ is locally support finite, then $A$ is $\nu_A$-exhaustive. Now, the following theorem given by Assem and Skowroński in [AS2], establishes a classification of the repetitive algebra $\widehat{\mathbf{A}}$ which are locally support finite.

**Theorem 2.1.** *Let $A$ be a $k$-algebra. The following conditions are equivalent:*

*i) $\widehat{\mathbf{A}}$ is tame and exhaustive.*

*ii) $\widehat{\mathbf{A}}$ is tame and locally support finite.*

*iii) There exist an algebra $B$ which is either tilted of Dynkin type,* *or representation-infinite tilted of Euclidean type, or tubular, such that* $\widehat{\mathbf{A}} \cong \widehat{B}$.

*iv) There exist an algebra* $\mathcal{C}$ *which is either hereditary of Dynkin or Euclidean type, or tubular canonical, such that* $A$ *and* $\mathcal{C}$ *are tilting-cotilting equivalent.*

*v)* $\underline{mod}\widehat{\mathbf{A}}$ *is cycle-finite.*

*vi) There exist an algebra* $\mathcal{C}$ *which is either hereditary of Dynkin or Euclidean type, or tubular canonic such that* $\underline{mod}\widehat{\mathbf{A}} \cong \underline{mod}\widehat{\mathcal{C}}$.

## 3. Representation of $T(A)$.

Let $Q$ be a quiver, $Sp$ be a subset of the loops of $Q$, and $\mathcal{R}$ be a set of relations for $Q$. We call the element of $Sp$ special loops, the remaining arrows are called ordinary. Let $\mathcal{R}^{Sp} := \{x^2 - x : x \in Sp\}$, and write $(\bar{\mathcal{R}})$ for the ideal in $kQ/(\mathcal{R}^{Sp})$ generated b imagen of the element, of $\mathcal{R}$ and denote $J$ the ideal of $kQ/(\mathcal{R}^{Sp})$ generated by the ordinary arrows.

A triple $(Q, Sp, \mathcal{R})$ as above is called clannish (see 2.5 in [C-B]) if the following conditions hold:

1) $(\bar{\mathcal{R}}) \subset J^2$

2) for any vertex of $Q$ at most 2 arrows start, and at most 2 arrows stop;

3) for every ordinary arrow $\beta$ there is at most one arrow $\alpha$ with $\alpha\beta \notin \mathcal{R}$, and at most one arrow $\gamma$ with $\beta\gamma \notin \mathcal{R}$.

We consider now the following lemma.

**Lemma 3.1.** *Let* $A$ *be a 2-tubular* $k$*-algebra. Then the trivial extension* $T(A)$ *is tame, and the category* $modT(A)$ *is equivalent to a category* $mod\mathcal{C}$, *where* $\mathcal{C}$ *is clannish.*

Proof. Let $A$ be a 2-tubular $k$-algebra. By lemma 2.1, we have that $\mathcal{D}^b(A) \underset{t}{\cong} \mathcal{D}^b(D)$, where $D$ is given by the quiver in figure 1.

Hence, the ordinary quiver of the trivial extension $T(D)$ is given by:

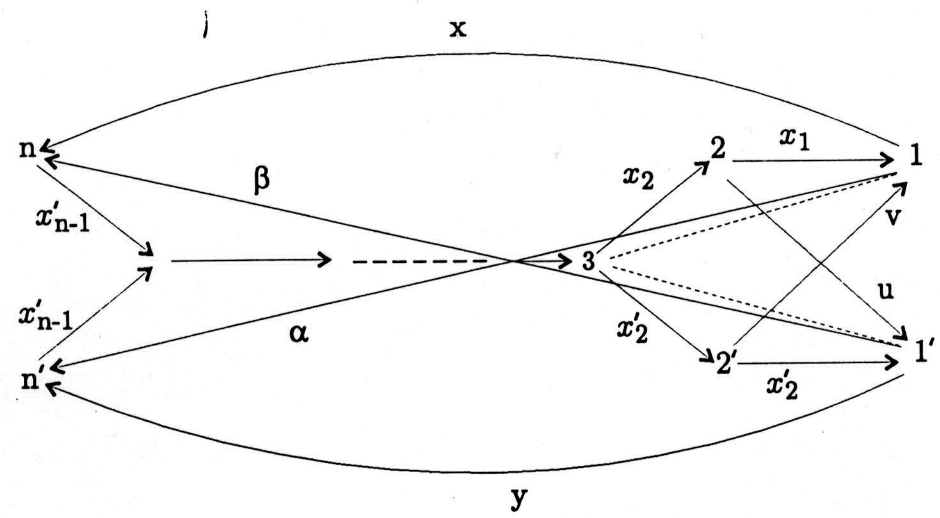


Figure 2.

with commutativity relations given by: $u\beta = x_1 x$, $v\alpha = x'_1 y$, $x x_{n-1} = \alpha x'_{n-1}$, $x_2 u = x'_2 x'_1$, $\beta x_{n-1} = y x'_{n-1}$, $x'_2 v = x_2 x_1$ We considered now, the following clannish algebra $\mathcal{C}$, given by the following quiver:

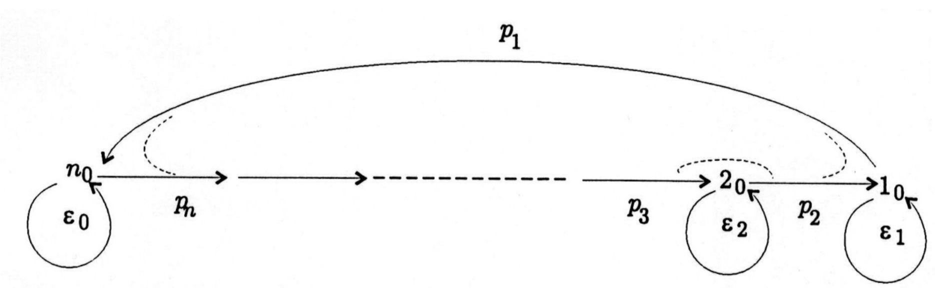


Figure 3.

where $q_{\varepsilon_i}(\varepsilon_i) = (\varepsilon_i - K_1^i)(\varepsilon_i - K_2^i) = 0$ with $K_1^i \neq K_2^i \in k^*$, where $i = 0, 1, 2$ and as zero relation we have: $p_1 p_n = p_3 p_2 = p_2 p_1 = 0$.

Now, each module $M \in mod\mathcal{C}$, has the form: $M(n_0) = M_{K_1^0} \oplus M_{K_2^0}$ where each $M_{K_i^0}$ is a $k$-vector space associated to the eigenvalue $K_i^0$ see 2.6 in [C-B].

Then we can defined a functor $F : modT(D) \longrightarrow mod\mathcal{C}$ in the following form: Let $X \in modT(D)$

$$F(X)(i_0) = \begin{cases} X_n \oplus X_n' & \text{if } \; i = n \\ X_2 \oplus X_2' & \text{if } \; i = 2 \\ X_1 \oplus X_1' & \text{if } \; i = 1 \\ X_i & \text{if } \; i \neq \{1, 2, n\} \end{cases}$$

$$F(X)(p_i) = \begin{cases} \begin{pmatrix} X(x) & X(\beta) \\ -X(\alpha) & -X(y) \end{pmatrix} & \text{if } \; i = 1 \\ \begin{pmatrix} X(x_1) & X(v) \\ -X(u) & -X(x_1') \end{pmatrix} & \text{if } \; i = 2 \\ \begin{pmatrix} X(x_2) \\ -X(x_2') \end{pmatrix} & \text{if } \; i = 3 \\ \begin{pmatrix} X(x_{n-1}) & X(x_{n-1}' \end{pmatrix} & \text{if } \; i = n \\ X(x_i) & \text{if } \; i \neq \{1, 2, 3, n\} \end{cases}$$

$$F(X)(\varepsilon_i) = \begin{cases} \begin{pmatrix} K_1^1 1_{x_1} & 0 \\ 0 & K_2^1 1_{x_1'} \end{pmatrix} & \text{if } \; i = 1 \\ \begin{pmatrix} K_1^2 1_{x_2} & 0 \\ 0 & K_2^2 1_{x_2'} \end{pmatrix} & \text{if } \; i = 2 \\ \begin{pmatrix} K_1^0 1_{x_n} & 0 \\ 0 & K_2^0 1_{x_n'} \end{pmatrix} & \text{if } \; i = n \end{cases}$$

We define now the functor $G : mod\mathcal{C} \longrightarrow modT(D)$. Let $M \in mod\mathcal{C}$, have that:

$$p_{i_0} = \begin{pmatrix} M_{11}^{i_0} & M_{12}^{i_0} \\ M_{21}^{i_0} & M_{22}^{i_0} \end{pmatrix}$$

where the $M_{tj}^{i_0} : M_{K_t^{i_0}} = M_t \longrightarrow M_{K_j^{i_0}} = M_j$ $i, t, j = 1, 2$, hence

$$p_n = \begin{pmatrix} M_{1n-1}^n & M_{2n-1}^n \end{pmatrix}, p_{n-1} = \begin{pmatrix} M_{n-1n-2}^{n-1} \end{pmatrix} \text{ and } \; p_3 = \begin{pmatrix} M_{31}^3 \\ M_{32}^3 \end{pmatrix}$$

We defined the functor $G$ in the following form: Let $M \in mod\mathcal{C}$

$$G(M)(x) = \begin{cases} M_2 & \text{if} \quad x = x_i, \quad i = 1, 2, n \\ M_1 & \text{if} \quad x = x_i^{'}, \quad i = 1, 2, n \\ M_j & \text{if} \quad x = x_j, \quad j \neq \{1, 2, n\} \end{cases}$$

$$\begin{array}{lcllcl} G(M)(x) & = & M_{22}^1(p_1) & G(M)(x_1) & = & M_{22}^2(p_2) \\ G(M)(\alpha) & = & M_{21}^1(p_1) & G(M)(u) & = & M_{21}^2(p_2) \\ G(M)(\beta) & = & -M_{12}^1(p_1) & G(M)(v) & = & -M_{12}^2(p_2) \\ G(M)(y) & = & -M_{11}^1(p_1) & G(M)(x_1^{'}) & = & -M_{11}^2(p_2) \\ G(M)(x_2) & = & M_{32}^3(p_3) & G(M)(x_{n-1}) & = & -M_{23}^n(p_n) \\ G(M)(x_2^{'}) & = & M_{31}^3(p_3) & G(M)(x_{n-1}^{'}) & = & M_{13}^n(p_3) \\ G(M)(x_j) & = & M_{j\ j+1}(p_j) & \text{if} & j \neq & \{1, 2, 3, n\} \end{array}$$

Hence, from the relations of Clannish algebra $\mathcal{C}$, is easy verify that this definition of the functor $G$, defined a module in $modT(D)$. By the construction these functors $F$ and $G$ we have that $F \circ G = 1_{modT(D)}$ and $G \circ F = 1_{mod\ \mathcal{C}}$. But, is known that an algebra Clannish is tame (see [C-B]), then we have that the trivial extension $T(D)$ is tame. Since, a derived equivalence induces a stable equivalence between the trivials extensions (see [HW]), then $\underline{mod}T(A) \cong \underline{mod}T(D)$ then by Krause (see [Kr]) we have that the trivial extension $T(A)$ is tame.

**Proposition 3.1 ( prop. 2 in [DS]).** *Let $R$ be a locally bounded $k$-category, and $G$ be the group of the $k$-linear automorphism of $R$ acting freely on the objects of $R$. If $R/G$ is tame, then $R$ also is tame.*

**Theorem 3.1.** *Let $A$ be a tame concealed algebra, $M$ be an indecomposable module in $modA$, and assume $B = A[M]$ is of tame representation. Then the trivial extension $T(B)$ is tame if and only if the module $M$ is regular.*

Proof. Assume $T(B)$ is tame. How, $T(B) \cong \widehat{B}/(\nu)$, then by Proposition 3.1, we have that $\widehat{B} = \widehat{A[M]}$ is tame. Moreover, by lemma 3 in Ringel [R2] if $A[M]$ is tame, then the module $M$ is regular or preinjective.

If the module $M$ is preinjective, it is known that $\widehat{A[M]} \cong \widehat{[M]A}$, and $[M]A$ is wild by lemma 3 in [R2], hence a full wild subcategory of $\widehat{[M]A}$. Thus, $\widehat{B}$ is wild,which is a contradiction. Therefore, the module $M$ is regular.

Assume $M$ is an indecomposable regular module. We consider two situations :

**a)** The algebra $A$ is not concealed of type $\widetilde{\mathbf{A_n}}$. Hence, we have that the algebra $B$ is tubular or 2-tubular (see 2.2 in [P]).

If $B$ is tubular, then by theorem 2.1 we have that $\widehat{B}$ is tame and exhautive, therefore the trivial extension $T(B)$ is tame. If $B$ is 2-tubular, then by lemma 3.1 we have that the trivial extension $T(B)$ is tame.

**b)** $A = \widetilde{\mathbf{A_n}}$. We use to the table given by Ringel (see th. 3 in [R2]), we have two situations:

**b1)** The module $M$ is homogeneus. In this situation we have:

1) $(\widetilde{\mathbf{A}_{22}}, 1)$; 2) $(\widetilde{\mathbf{A}_{23}}, 1)$; 3) $(\widetilde{\mathbf{A}_{24}}, 1)$; 4) $(\widetilde{\mathbf{A}_{25}}, 1)$; 5) $(\widetilde{\mathbf{A}_{26}}, 1)$; 6) $(\widetilde{\mathbf{A}_{2q}}, 1)$ if $q \geq 7$

7) $(\widetilde{\mathbf{A}_{33}}, 1)$; 8) $(\widetilde{\mathbf{A}_{34}}, 1)$; 9) $(\widetilde{\mathbf{A}_{35}}, 1)$; 10)$(\widetilde{\mathbf{A}_{36}}, 1)$;

11)$(\widetilde{\mathbf{A}_{44}}, 1)$

In the situation 1) to 6), these algebras are domestic tubular of type (2,2,q) and that corresponds to the tubular type of $\mathbf{D}_{q+2}$. In the situation 7) at 9), this algebras are domestic tubular of type (2,3,3), (2,3,4) and (2,3,5) which corresponds to $\mathbf{E}_6$, $\mathbf{E}_7$ and $\mathbf{E}_8$ respectively. Now, the situation 10) and 11) is also tubular of type (2,3,6) (2,4,4) corresponding to euclidean type $\widetilde{\mathbf{E}}_8$ and $\widetilde{\mathbf{E}}_7$ respectively. By theorem 2.1 we have that the trivial extension $T(B)$ is tame.

**b2)**The module $M$ is not homogeneus. Hence, we have two cases:

**b2.1)** $(\widetilde{\mathbf{A}_{pq}}, p)$, where the module $M$ lies in the mouth of tube of rank $p$. Hence, $(\widetilde{\mathbf{A}_{p+1q}})$ which is tubular, therefore the trivial extension $T(B)$ is tame.

**b2.2)** $(\widetilde{\mathbf{A_{pq}}}, 2p)$, where the module $M$ lies in a tube of rank $p$, and regular of length 2. It is easy to see that the proof of theorem 3 in Ringel [R2] if $\widetilde{\mathbf{A_{pq}}}[N]$ with module $N$ has regular length 2 there exist a sequence of reflextions which takes $B$ into $\widetilde{\mathbf{A_{pq}}}[N]$, then $\mathcal{D}^b\,(B)\; \underset{t}{group}$ $\cong \mathcal{D}^b\left(\widetilde{\mathbf{A_{pq}}}[\mathbf{N}]\right)$.

We consider the $C$ algebra, given by the quiver in the following figure:

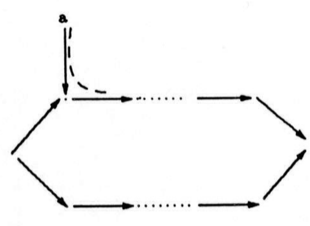


Figure 4.

where the dotted lines is zero relation. We have that $C := \widetilde{\mathbf{A}_{p-1q}}[M]$ where the module $M$ is defined by:

$$M := \begin{array}{cccccc} & 1 & 0 & \dots & & \\ 0 & & & & & 0 \\ & 0 & \dots & 0 & & \end{array}$$

that is lies in the mouth the tube of rank $p-1$, the algebra $C$ is a tilted algebra of type $\widetilde{\mathbf{A}_{pq}}$ then there exist a functor $\rho : \mathcal{D}^b(C) \longrightarrow \mathcal{D}^b(\widetilde{\mathbf{A}_{pq}})$, that give a triangular equivalence such that $\rho(M)$ lyies in the tube of rank $p$, and is a regular of lenght 2.

We consider now the algebra $D$ defined by the following quiver:

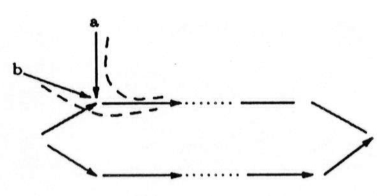


Figure 5.

Here the module $M$ is the same as before. Then, by Barot-Lenzing (see [BL]), we have that: $\mathcal{D}^b(D) \underset{t}{\cong} \mathcal{D}^b((\widetilde{\mathbf{A}_{pq}})[\rho(M)])$, where $\widetilde{\mathbf{A}_{pq}}[\rho(M)]$ is a one-point extension by module $\rho(M)$ of regular length 2, then $\mathcal{D}^b(B) \underset{t}{\cong} \mathcal{D}^b((\widetilde{\mathbf{A}_{pq}})[\rho(M)])$. Using the similar construction as the lemma 3.1 with clannish algebras is not dificult to see that $modD$ is equivalent to a clannish algebra $E$ given by the following quiver:

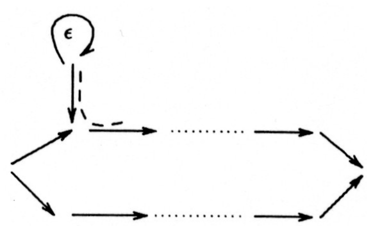


Figure 6.

Where $\epsilon$ is special loop. By Geiss and De la Peña (see 4.4 in [GEP]) the trivial extension $T(E)$ is tame, thus $T(D)$ is tame. Since $\mathcal{D}^b(D) \underset{t}{\cong} \mathcal{D}^b(B)$, therefore $\underline{mod}T(D) \cong \underline{mod}T(B)$, then by [Kr] we have that $T(B)$ is tame.

Before to state the next result, we consider $A \cong kQ/I$ be an algebra, where $Q$ is a quiver without oriented cycles. Let $\chi_A : \mathbf{Z}^{Q_0} \longrightarrow \mathbf{Z}$ and $q_A : \mathbf{Z}^{Q_0} \longrightarrow \mathbf{Z}$ be the quadratic forms defined by:

$$\begin{aligned} \chi_A(v) &= \sum_{s=0}^{\infty} \sum_{i,j \in Q_0} (-1)^s Ext_A^s(S_i, S_j) v_i v_j \\ q_A(v) &= \sum_{i \in Q} v_i^2 - \sum_{(i \to j) \in Q_1} v_i v_j + \sum_{i,j \in Q_0} r(i,j) v_i v_j \end{aligned}$$

where $v = (v_1, ..., v_n)$ , $r(i,j) = dim_k e_j(I/(IJ + JI))e_i$, and $J$ is the ideal generated by arrows of the quiver $Q$. The quadratic form $\chi_A$ is called the Euler form of the algebra $A$, and $q_A$ is called the Tits form. Its know that if $gldimA \leq 2$, then $\chi_A = q_A$.

As the consequence of our theorem 3.1, we have the following corollary.

**Corollary 3.1.** *Let $A \cong kQ/I$, where $Q$ is a tree, such that Euler form $\chi_A$ is non-negative and corank$\chi_A \leq 2$. Then the tivial extension $T(A)$ is tame.*

Proof. By Barot-De la Peña (see [BP]) we have that the algebra $A$ is domestic tubular, tubular or 2-tubular. Therefore, by the above theorem we have that the trivial extension $T(A)$ is tame.

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