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SEQUENTIAL S^* -COMPACTNESS IN L -TOPOLOGICAL SPACES *

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Abstract

In this paper, a new notion of sequential compactness is introduced in L -topological spaces, which is called sequentially S^ -compactness. If $L = [0, 1]$, sequential ultra-compactness, sequential N -compactness and sequential strong compactness imply sequential S^* -compactness, and sequential S^* -compactness implies sequential F -compactness. The intersection of a sequentially S^* -compact L -set and a closed L -set is sequentially S^* -compact. The continuous image of an sequentially S^* -compact L -set is sequentially S^* -compact. A weakly induced L -space (X, T) is sequentially S^* -compact if and only if $(X, [T])$ is sequential compact. The countable product of sequential S^* -compact L -sets is sequentially S^* -compact.*

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1. Introduction and Preliminaries

In [6], the notion of S^* -compactness was presented in L -topological spaces by means of β_a -cover and Q_a -cover. It can be characterized in terms of constant a -nets and their weak O-cluster points. If $L = [0, 1]$, then strong compactness implies S^* -compactness and S^* -compactness implies Lowen's compactness.

In this paper, based on the notion of S^* -compactness, we shall introduce the notion of sequential S^* -compactness by constant a -sequences and their weak O-limit points. We shall research its properties and the relation between it and other sequential compactness.

Throughout this paper $(L, \vee, \wedge, ')$ is a completely distributive de Morgan algebra, X is a nonempty set. L^X is the set of all L -fuzzy sets on X . The smallest element and the largest element in L^X are denoted by $\underline{0}$ and $\underline{1}$.

An element a in L is called prime if $a \geq b \wedge c$ implies $a \geq b$ or $a \geq c$. An element a in L is called co-prime if a' is a prime element [3]. The set of nonunit prime elements in L is denoted by $P(L)$. The set of nonzero co-prime elements in L is denoted by $M(L)$. The set of nonzero co-prime elements in L^X is denoted by $M(L^X)$.

The binary relation $\prec$ in L is defined as follows: for $a, b \in L$, $a \prec b$ if and only if for every subset $D \subseteq L$, the relation $b \leq \sup D$ always implies the existence of $d \in D$ with $a \leq d$ [2]. In a completely distributive de Morgan algebra L , each member b is a sup of $\{a \in L \mid a \prec b\}$. In the sense of [4, 9], $\{a \in L \mid a \prec b\}$ is the greatest minimal family of b , in symbol $\beta(b)$. Moreover for $b \in L$, define $\alpha(b) = \{a \in L \mid a' \prec b'\}$ and $\alpha^*(b) = \alpha(b) \cap P(L)$.

For an L -set $A \in L^X$, $\beta(A)$ denotes the greatest minimal family of A and $\beta^*(A) = \beta(A) \cap M(L^X)$.

For $a \in L$ and $A \in L^X$, we use the following notations in [6].

$$A_{(a)} = \{x \in X \mid a \in \beta(A(x))\}, \quad A^{(a)} = \{x \in X \mid A(x) \not\leq a\}.$$

An L -topological space (or L -space for short) is a pair $(X, \mathcal{T})$, where $\mathcal{T}$ is a subfamily of L^X which contains $\underline{0}$, $\underline{1}$ and is closed for any suprema and finite infima. $\mathcal{T}$ is called an L -topology on X . Each member of $\mathcal{T}$ is called an open L -set and its complement is called a closed L -set.

Definition 1.1 ([4, 9]). For a topological space (X, τ) , let $\omega_L(\tau)$ denote the family of all the lower semi-continuous maps from (X, τ) to L , i.e.,

$\omega_L(\tau) = \{A \in L^X \mid A^{(a)} \in \tau, a \in L\}$. Then $\omega_L(\tau)$ is an L -topology on X , in this case, $(X, \omega_L(\tau))$ is called topologically generated by (X, τ) .

Definition 1.2 ([4, 9]). An L -space $(X, \mathcal{T})$ is called weak induced if $\forall a \in L, \forall A \in \mathcal{T}$, it follows that $A^{(a)} \in [\mathcal{T}]$, where $[\mathcal{T}]$ denotes the topology formed by all crisp sets in $\mathcal{T}$.

Lemma 1.3 ([6]). Let $(X, \mathcal{T})$ be a weakly induced L -space, $a \in L, A \in \mathcal{T}$. Then $A_{(a)}$ is an open set in $[\mathcal{T}]$.

Ultra-sequential compactness, N -sequential compactness, strong sequential compactness and sequential compactness had be generalized to $[0, 1]$ -topological space by L.X. Xuan in [10, 11, 12, 13].

Definition 1.4 ([10]). A $[0, 1]$ -space $(X, \mathcal{T})$ is called sequentially ultra-compact or ultra-sequentially compact if $\iota_L(\mathcal{T})$ is sequentially compact, where $\iota(\mathcal{T})$ is the topology generated by $\{A^{(a)} \mid A \in \mathcal{T}, a \in [0, 1]\}$.

Definition 1.5 ([4, 9]). Let $a \in M(L)$. A net $\{S(n) \mid n \in D\}$ in L^X is said to be constant a -net if the height $V(S(n))$ of each $S(n)$ is a constant value a . $\{S(n) \mid n \in D\}$ is said to be an a -net if for each $r \in \beta^*(a)$, there exists $n(r) \in D$ such that $V(S(n)) \geq r$ for all $n \geq n(r)$.

Definition 1.6 ([7]). A net S with index set D in L^X is also denoted by $\{S(n) \mid n \in D\}$ or $\{S(n)\}_{n \in D}$. For $G \in L^X$, a net S is said to quasi-coincide with G if $\forall n \in D, S(n) \not\leq G'$.

Definition 1.7 ([4, 9]). Let $\{S(n) \mid n \in D\}$ be a net in $(X, \mathcal{T})$, $x_\lambda \in M(L^X)$. x_λ is called a cluster point of S if for each closed R -neighborhood P of x_λ , S is not frequently in P . x_λ is called a limit point of S if for each closed R -neighborhood P of x_λ , S is not eventually in P , in this case we also say that S converges to x_λ .

Definition 1.8 ([11]). A $[0, 1]$ -space $(X, \mathcal{T})$ is called sequentially N -compact or N -sequentially compact if for all $a \in (0, 1]$, each a -sequence has a cluster point x_a .

Definition 1.9 ([12]). A $[0, 1]$ -space $(X, \mathcal{T})$ is called sequentially strongly compact or strongly sequentially compact if for all $a \in (0, 1]$, each constant a -sequence has a cluster point x_a .

Definition 1.10 ([13]). A $[0, 1]$ -space $(X, \mathcal{T})$ is called sequentially F -compact or fuzzy sequentially compact if for all $a \in (0, 1]$ and all $b \in (0, a)$, each constant a -sequence has a cluster point x_b .

Definition 1.11 ([6]). Let $(X, \mathcal{T})$ be an L -space. An open L -set U is called a strong open neighborhood of a fuzzy point x_λ , if $\lambda \in \beta(U(x))$.

Definition 1.12 ([6]). Let $\{S(n) \mid n \in D\}$ be a net in $(X, \mathcal{T})$, $x_\lambda \in M(L^X)$. x_λ is called a weak O -cluster point of S , if for each strong open neighborhood U of x_λ , S is frequently in U . x_λ is called a weak O -limit point of S , if for each strong open neighborhood U of x_λ , S is eventually in U , in this case we also say that S weakly O -converges to x_λ , denoted by $S \xrightarrow{WO} x_\lambda$.

2. Sequentially S^* -compactness

Definition 2.1. An L -set G in $(X, \mathcal{T})$ is called sequentially S^* -compact if for all $a \in M(L)$, each constant a -sequence quasi-coinciding with G has a subsequence which weak O -converges to $x_a \notin \beta(G')$

Theorem 2.2. An L -set with finite support is sequentially S^* -compact.

Proof. Suppose that G is an L -set with finite support in $(X, \mathcal{T})$ and $\{x_a^n \mid n \in \mathbf{N}\}$ is a constant a -sequence quasi-coinciding with G . Then $\{x_a^n \mid n \in \mathbf{N}\}$ has a subsequence $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$ such that $x_a^{n(k)} = x_a^{n(1)}$ for all $k \in \mathbf{N}$. In this case, $x_a^{n(1)}$ is a weak O -limit point of $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$ and $x_a^{n(1)} \notin \beta(G')$

Definition 2.3. An L -space $(X, \mathcal{T})$ is said to be strong C_I , if for each $x_\lambda \in M(L^X)$, there exists a countable local base of strong open neighborhoods of x_λ .

It is easy to prove the following lemma.

Lemma 2.4. An L -space $(X, \mathcal{T})$ is strong C_I if and only if for each $x_\lambda \in M(L^X)$, there exists a countable local base $\{U_i \mid i \in \mathbf{N}\}$ of strong open neighborhoods of x_λ such that $U_1 \supset U_2 \supset \cdots$.

Theorem 2.5. Let $(X, \mathcal{T})$ be a strong C_I L -space. Then $G \in L^X$ is sequentially S^* -compact if and only if each constant a -net quasi-coinciding with G has a weak O -cluster point $x_a \notin \beta(G')$.

Proof. The necessity is obvious. Now we prove the sufficiency. Suppose that $\{x_a^n \mid n \in \mathbf{N}\}$ is a constant a -sequence quasi-coinciding with G and it has a weak O -cluster point $x_a \notin \beta(G')$. Let $\{U_i \mid i \in \mathbf{N}\}$ be a countable local base at x_a such that $U_1 \supset U_2 \supset \cdots$. Then for each $k \in \mathbf{N}$, there exists an $n(k) \geq k$ such that $x_a^{n(k)} \leq U_k$. It is easy to see that $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$ weak O -converges to x_a . Therefore $G \in L^X$ is sequentially S^* -compact.

By Theorem 2.5 and analogous to the proof of Theorem 5.3 in [6], we can obtain the following result.

Theorem 2.6. Let $(X, \mathcal{T})$ be a C_{II} L -space. Then $G \in L^X$ is sequentially S^* -compact if and only if it is S^* -compact.

Theorem 2.7. If G is sequentially S^* -compact and H is closed, then $G \wedge H$ is sequentially S^* -compact.

Proof. Suppose that $\{x_a^n \mid n \in \mathbf{N}\}$ is a constant a -sequence quasi-coinciding with $G \wedge H$. Then $\{x_a^n \mid n \in \mathbf{N}\}$ is also quasi-coincident with G . By sequential S^* -compactness of G we know that $\{x_a^n \mid n \in \mathbf{N}\}$ has a subsequence $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$ weak O -converging to $x_a \notin \beta(G')$. We can prove that $x_a \notin \beta(H')$ since $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$ is quasi-coincident with H . Therefore $x_a \notin \beta(G' \vee H')$.

Theorem 2.8. *If G is sequentially S^* -compact in $(X, \mathcal{T}_1)$ and $f : (X, \mathcal{T}_1) \rightarrow (Y, \mathcal{T}_2)$ is continuous, then $f_L^\rightarrow(G)$ is sequentially S^* -compact in $(Y, \mathcal{T}_2)$.*

Proof. Suppose that $\{y_a^n \mid n \in \mathbf{N}\}$ is a constant a -sequence quasi-coinciding with $f_L^\rightarrow(G)$. Then there exists a constant a -sequence $\{x_a^n \mid n \in \mathbf{N}\}$ quasi-coinciding with G such that $f(x^n) = y^n$ for all $n \in \mathbf{N}$. By sequential S^* -compactness of G we know that $\{x_a^n \mid n \in \mathbf{N}\}$ has a subsequence $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$ weak O-converging to $x_a \notin \beta(G')$. It is easy to prove that $\{y_a^{n(k)} \mid k \in \mathbf{N}\}$ weak O-converges to $f(x)_a$ and $f(x)_a \notin \beta(f(G'))$. Therefore $f_L^\rightarrow(G)$ is sequentially S^* -compact.

Theorem 2.9. *If $(X, \mathcal{T})$ is a weakly induced L -space, then $(X, \mathcal{T})$ is sequentially S^* -compact if and only if $(X, [\mathcal{T}])$ is sequentially compact.*

Proof. Let $(X, [\mathcal{T}])$ be sequentially compact. For $a \in M(L)$, let $\{x_a^n \mid n \in \mathbf{N}\}$ be a constant a -sequence quasi-coinciding with $\underline{1}$ in $(X, \mathcal{T})$. Then $\{x^n \mid n \in \mathbf{N}\}$ is a sequence in $(X, [\mathcal{T}])$. By sequential compactness of $(X, [\mathcal{T}])$, we know that $\{x^n \mid n \in \mathbf{N}\}$ has a subsequence $\{x^{n(k)} \mid k \in \mathbf{N}\}$ converging to x . Now we prove that $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$ weak O-converges to x_a in $(X, \mathcal{T})$. In fact, let U is a strong open neighborhood of x_a . By Lemma 1.3 we know that $U_{(a)}$ is an open neighborhood of x in $[\mathcal{T}]$. Thus $\{x^{n(k)} \mid k \in \mathbf{N}\}$ is eventually in $U_{(a)}$. Hence $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$ is eventually in U . It is proved that $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$ weak O-converges to x_a . This shows that $(X, \mathcal{T})$ is sequentially S^* -compact.

Conversely let $(X, \mathcal{T})$ be sequentially S^* -compact and $\{x^n \mid n \in \mathbf{N}\}$ be a sequence in $(X, [\mathcal{T}])$. Then for each $a \in \beta^*(1)$, $\{x_a^n \mid n \in \mathbf{N}\}$ is a constant a -sequence quasi-coinciding with $\underline{1}$ in $(X, \mathcal{T})$. By sequential S^* -compactness of $(X, \mathcal{T})$, we know that $\{x_a^n \mid n \in \mathbf{N}\}$ has a subsequence $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$ weak O-converging to x_a in $(X, \mathcal{T})$. It is easy to prove that $\{x^{n(k)} \mid k \in \mathbf{N}\}$ converges to x in $(X, [\mathcal{T}])$. Therefore $(X, [\mathcal{T}])$ is sequentially compact.

Corollary 2.10. *Let (X, τ) be a crisp topological space. Then $(X, \omega_L(\tau))$ is sequentially S^* -compact if and only if (X, τ) is sequentially compact.*

Theorem 2.11. *Let $(X, \mathcal{T})$ be the countable product of a family of L -space's $\{(X_i, \mathcal{T}_i)\}_{i \in \mathbf{N}}$. If for each $i \in \mathbf{N}$, G_i is sequentially S^* -compact in $(X_i, \mathcal{T}_i)$, then $G = \prod_{i \in \mathbf{N}} G_i$ is sequentially S^* -compact in $(X, \mathcal{T})$.*

Proof. Let p_n denote the projection from X onto X_n . For $a \in M(L)$, let $\{x(i)_a \mid i \in \mathbf{N}\}$ be a constant a -sequence quasi-coinciding with G . Then for any $n \in \mathbf{N}$, $\{p_n(x(i))_a \mid i \in \mathbf{N}\}$ is a constant a -sequence quasi-coinciding with G_n . By sequential S^* -compactness of G_1 , we can choose a subsequence $\{x^1(i)_a \mid i \in \mathbf{N}\}$ of $\{x(i)_a \mid i \in \mathbf{N}\}$ such that $\{p_1(x^1(i)_a) \mid i \in \mathbf{N}\}$ weak O-converges to a point $y_a^1 \notin \beta((G_1)')$. In this case $\{p_2(x^1(i)_a) \mid i \in \mathbf{N}\}$ is a constant a -sequence quasi-coinciding with G_2 . By sequential S^* -compactness of G_2 , we can again choose a subsequence $\{x^2(i)_a \mid i \in \mathbf{N}\}$ of $\{x^1(i)_a \mid i \in \mathbf{N}\}$ such that $\{p_2(x^2(i)_a) \mid i \in \mathbf{N}\}$ weak O-converges to a point $y_a^2 \notin \beta((G_2)')$. In this case $\{p_3(x^2(i)_a) \mid i \in \mathbf{N}\}$ is a constant a -sequence quasi-coinciding with G_3 . $\dots$, by reduction for all $k \in \mathbf{N}$, we can obtain a subsequence $\{x^{k+1}(i)_a \mid i \in \mathbf{N}\}$ of $\{x^k(i)_a \mid i \in \mathbf{N}\}$ such that $\{p_{k+1}(x^{k+1}(i)_a) \mid i \in \mathbf{N}\}$ weak O-converges to a point $y_a^{k+1} \notin \beta((G_{k+1})')$. Take a sequence $\{x^i(i)_a \mid i \in \mathbf{N}\}$. Then $\{x^i(i)_a \mid i \in \mathbf{N}\}$ is a subsequence of $\{x(i)_a \mid i \in \mathbf{N}\}$. It is easy to see that $\{x^i(i)_a \mid i \in \mathbf{N}\}$ becomes a subsequence of $\{x^k(i)_a \mid i \in \mathbf{N}\}$ after dropping first $k - 1$ terms.

Let $y_a = \prod_{k \in \mathbf{N}} y_a^k$. Then $y_a \notin \beta(G')$. In fact, if $y_a \in \beta(G')$, then by

$$G = \prod_{n \in \mathbf{N}} G_n = \bigwedge_{n \in \mathbf{N}} p_n^{-1}(G_n)$$

we obtain that $y_a \in \beta\left(\bigvee_{n \in \mathbf{N}} p_n^{-1}(G_n)'\right)$, hence there is $n \in \mathbf{N}$ such that $y_a \in \beta(p_n^{-1}(G_n)')$, this implies that $y_a^n \in \beta((G_n)')$, we obtain a contradiction.

Now we prove that y_a is a weak O-limit point of $\{x^i(i)_a \mid i \in \mathbf{N}\}$. Suppose that U is a strong open neighborhood of y_a in $\mathcal{T}$. Then there exists a finite subset T of $\mathbf{N}$ and open set $U_n \in \mathcal{T}_n$ for each $n \in T$ such that

$\bigwedge_{n \in T} p_n^{-1}(U_n) \leq U$ and $y_a \in \beta\left(\bigwedge_{n \in T} p_n^{-1}(U_n)\right) \subset \beta(U)$. Hence for all $n \in T$, $y_a^n = p_n(y_a) \in \beta(U_n)$. Thus $\{p_n(x^i(i)_a) \mid i \in \mathbf{N}\}$ is eventually in U_n for each $n \in T$. This implies that $\{x^i(i)_a \mid i \in \mathbf{N}\}$ is eventually in $\bigwedge_{n \in T} p_n^{-1}(U_n) \leq U$.

Therefore y_a is a weak O-limit point of $\{x^i(i)_a \mid i \in \mathbf{N}\}$. The proof is completed.

From Theorem 2.8 and Theorem 2.11 we can obtain the following corollary.

Corollary 2.12. *Let $(X, \mathcal{T})$ be the countable product of a family of L -space's $\{(X_i, \mathcal{T}_i)\}_{i \in \mathbf{N}}$. If for each $i \in \mathbf{N}$, $(X_i, \mathcal{T}_i)$ is sequentially S^* -compact, then so is $(X, \mathcal{T})$. Conversely if $(X, \mathcal{T})$ is sequentially S^* -compact and $(X_i, \mathcal{T}_i)$ is fully stratified, then $(X_i, \mathcal{T}_i)$ is sequentially S^* -compact.*

3. A comparison of a few kinds of sequential compactness

In order to compare different notions of fuzzy sequential compactness, we only consider sequential S^* -compactness of $[0, 1]$ -topological spaces.

Theorem 3.1. *For a $[0, 1]$ -space, sequential ultra-compactness, sequential N -compactness and sequential strong compactness imply sequential S^* -compactness, and sequential S^* -compactness implies sequential F -compactness.*

Proof. We only need to prove that sequential strong compactness implies sequential S^* -compactness and sequential S^* -compactness implies sequential F -compactness. Let $(X, \mathcal{T})$ be a sequentially strong compact $[0, 1]$ -space. To prove that it is sequentially S^* -compact, suppose that $\{x_a^n \mid n \in \mathbf{N}\}$ is a constant a -sequence in $(X, \mathcal{T})$. If $a = 1$, then for all $x \in X$, x_1 can be regard as a weak O-limit point of $\{x_a^n \mid n \in \mathbf{N}\}$ since none of open fuzzy sets is strong open neighborhood of x_1 . Now we suppose that $a \in (0, 1)$. Take $b = 1 - a$. Then $\{x_b^n \mid n \in \mathbf{N}\}$ is a constant b -sequence. By sequential strong compactness of $(X, \mathcal{T})$, we know that $\{x_b^n \mid n \in \mathbf{N}\}$ has a subsequence $\{x_b^{n(k)} \mid k \in \mathbf{N}\}$ converging to a point x_b . We shall prove that x_a is a weak O-limit point of $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$. In fact, for each strong open neighborhood U of x_a , U is an open quasi-neighborhood of x_b . Since $\{x_b^{n(k)} \mid k \in \mathbf{N}\}$ converges to x_b , we know that $\{x_b^{n(k)} \mid k \in \mathbf{N}\}$ is eventually quasi-coincident with U . This implies that $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$ is eventually in U . Therefore x_a is a weak O-limit point of $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$. Sequential S^* -compactness of $(X, \mathcal{T})$ is proved.

Further let $(X, \mathcal{T})$ be a sequentially S^* -compact $[0, 1]$ -space. To prove that it is sequentially F -compact, suppose that $\{x_a^n \mid n \in \mathbf{N}\}$ is a constant a -sequence, where $a \in (0, 1]$. For any $r \in (0, a)$, let $b = 1 - r$, then $\{x_b^n \mid n \in \mathbf{N}\}$ is a constant b -sequence. By sequential S^* -compactness of $(X, \mathcal{T})$, we know that $\{x_b^n \mid n \in \mathbf{N}\}$ has a subsequence $\{x_b^{n(k)} \mid k \in \mathbf{N}\}$ weak O-converging to a point x_b . We shall prove that x_r is a limit point of $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$. In fact, for each open quasi-neighborhood U of x_r , U is a

strong open neighborhood of x_b . Since $\{x_b^{n(k)} \mid k \in \mathbf{N}\}$ weak O-converges to x_b , we know that $\{x_b^{n(k)} \mid k \in \mathbf{N}\}$ is eventually in U . This implies that $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$ is eventually quasi-coincident with U . Therefore x_r is a limit point of $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$. Sequential F-compactness of $(X, \mathcal{T})$ is proved.

In general, each inverse in Theorem 4.1 needn't be true. This can be seen from the following two examples.

Example 3.2. Take $X = \mathbf{N}$. $\forall n \in X$, take $B_n \in [0, 1]^X$ as follows:

$$B_n(x) = \begin{cases} 0.5 + \frac{1}{n+1}, & x = n \\ 0.5 - \frac{1}{n+1}, & x \neq n \end{cases}$$

Let $\mathcal{T}$ be the $[0, 1]$ -topology generated by the subbase $\mathcal{B} = \{B_n \mid n \in \mathbf{N}\}$. As proved in [6], $(Y, \mathcal{T})$ is Lowen's fuzzy compact but it not S^* -compact. It is easy to see that $(Y, \mathcal{T})$ is a $C_{\mathcal{I}}$ $[0, 1]$ -space. Therefore it is sequentially F-compact but not sequentially S^* -compact.

Example 3.3. Take $Y = \mathbf{N}$. $\forall n \in \mathbf{N}$, take $B_n \in [0, 1]^Y$ as follows:

$$B_n(y) = \begin{cases} \frac{1}{n+1} + 0.5, & y = n, \\ 0.5, & y \neq n. \end{cases}$$

Let $\mathcal{T}$ be the topology generated by the subbase $\mathcal{B} = \{B_n \mid n \in \mathbf{N}\}$. As proved in [6], $(Y, \mathcal{T})$ is S^* -compact but it not strong compact. It is easy to see that $(Y, \mathcal{T})$ is a $C_{\mathcal{I}}$ $[0, 1]$ -space. Therefore it is sequentially S^* -compact but not sequentially strong compact.

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