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FUZZY PARA - LINDELOF SPACES

T. BAIJU

and

SUNIL JACOB JOHN

NATIONAL INSTITUTE OF TECHNOLOGY CALICUT, INDIA

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Abstract

In this paper we introduce the concept of Para-Lindelof spaces in L -topological spaces by means of locally countable families of L -fuzzy sets. Further some characterizations of fuzzy para-Lindelofness and flintily para-Lindelofness in the weakly induced L -topological spaces are also obtained. More over the behavior of fuzzy para-Lindelof spaces under various types of maps such as fuzzy closed maps, fuzzy perfect maps are also investigated.

Keywords : *L -Topology, Fuzzy para-Lindelofness, Flintily para-Lindelofness, locally countable family.*

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1. Introduction

As a generalization of a set, the concept of fuzzy set was introduced by Zadeh [18]. Fuzzy topology comes as the generalization of general topology using the concept of a fuzzy set. In 1968 Chang [6] introduced the concept of fuzzy topology and Lowen [12] introduced a more natural definition of fuzzy topology.

Compactness and metrizability are the heart and soul of general topology. In 1944 J. Dieudonne [7] defined paracompactness as a natural generalization of compactness. Later several other covering properties such as meta-compactness, sub para-compactness, sub meta-compactness, para-Lindelofness etc. have naturally evolved from para compactness. The concept of para-Lindelof spaces was introduced by J. Greever [9] in 1968 and further studies were conducted by Burke ([4, 5]), Fleissner-Reed [8].

The concept of paracompactness in fuzzy topology was introduced by Luo [13]. Authors have introduced the concept and studied some properties regarding metacompactness, subparacompactness, and submetacompactness in L -topological spaces in [14], [3], [2] respectively. In this paper we define locally countable families and introduce the concept of para-Lindelof spaces in L -topological spaces. Besides getting some characterization for para-Lindelof and flintily para-Lindelof in the weakly induced L -topological spaces, it is also seen that these properties are closed hereditary. Further the invariance of these properties under perfect maps is also proved.

Let L be a complete lattice. Its universal bounds are denoted by $\perp$ and $\top$. We presume that L is consistent. i.e., $\perp$ is distinct from $\top$. Thus $\perp \leq \alpha \leq \top$ for all $\alpha \in L$. We note $\vee \phi = \perp$ and $\wedge \phi = \top$. The two point lattice $\{\perp, \top\}$ is denoted by 2. A unary operation $'$ on L is a quasi-complementation. It is an involution (ie., $\alpha'' = \alpha$ for all $\alpha \in L$) that inverts the ordering. (ie., $\alpha \leq \beta$ implies $\beta' \leq \alpha'$). In $(L, ')$ the DeMorgan laws hold: $(\vee A)' = \wedge \{\alpha' : \alpha \in A\}$ and $(\wedge A)' = \vee \{\alpha' : \alpha \in A\}$ for every $A \subset L$. Moreover, in particular, $\perp' = \top$ and $\top' = \perp$.

A molecule or co-prime element in a lattice L is a join irreducible element in L and the set of all non zero co-prime elements of L is denoted by $M(L)$ and prime elements by $pr(L)$. A complete lattice L is completely distributive if it satisfies either of the logically equivalent CD1 or CD2 below: CD1: $\wedge_{i \in I} (\vee_{j \in J_i} a_{i,j}) = \vee_{\phi \in \prod J_i, i \in I} (\wedge_{i \in I} a_{i, \phi(i)})$

CD2: $\vee_{i \in I} (\wedge_{j \in J_i} a_{i,j}) = \wedge_{\phi \in \prod J_i, i \in I} (\vee_{i \in I} a_{i, \phi(i)})$

for all $\{\{a_{ij} : j \in J_i\} : i \in I\} \subset P(L) \setminus \{\phi\}$,

If L is a complete lattice, then for a set X , L^X is the complete lattice of

all maps from X into L , called L -sets or L -subsets of X . Under point-wise ordering, $a \leq b$ in L^X if and only if $a(x) \leq b(x)$ in L for all $x \in X$. If $A \subset X$, $1_A \in 2^X \subset L^X$ is the characteristic function of A . The constant member of L^X with value α is denoted by α itself. Usually we will not distinguish between a crisp set and its characteristic function. Wang [15] proved that a complete lattice is completely distributive if and only if for each $\alpha \in L$, there exists $B \subseteq L$ such that (i) $\alpha = \bigvee B$ and (ii) if $A \subseteq B$ and $\alpha \leq \bigvee A$, then for each $b \in B$, there exists $a \in A$ such that $b \leq a$. B is called the minimal set of α and $\beta(a)$ denote the union of all minimal sets of a . Again $\beta^*(a) = \beta(a) \cap M(L)$. Clearly $\beta(a)$ and $\beta^*(a)$ are minimal sets of a .

For $\alpha \in L$ and $A \in L^X$, we use the following notations.

$$\begin{aligned} A_{[\alpha]} &= \{x \in X : A(x) \geq \alpha\}; \\ A^{[\alpha]} &= \{x \in X : A(x) \leq \alpha\}; \\ A^{(\alpha)} &= \{x \in X : A(x) \not\geq \alpha\}; \\ A_{(\alpha)} &= \{x \in X : A(x) \not\leq \alpha\}. \end{aligned}$$

Clearly L^X has a quasi complementation $'$ defined point-wisely $\alpha'(x) = \alpha(x)'$ for all $\alpha \in L$ and $x \in X$. Thus the DeMorgan laws are inherited by $(L^X, ')$.

Let $(L, ')$ be a complete lattice equipped with an order reversing involution and X be any non empty set. A subfamily $\tau \subset L^X$ which is closed under the formation of sups and finite infs (both formed in L^X) is called an L -topology on X and its members are called open L -sets. The pair (X, τ) is called an L -topological space (L -ts). The category of all L -topological spaces, together with L -continuous mappings and the composition and identities of set is denoted by L -Top. Quasi complements of open L -sets are called closed L -sets.

We know that the set of all non zero co-prime elements in a completely distributive lattice is $\bigvee$ -generating. Moreover for a continuous lattice L and a topological space (X, T) , $T = i_L \omega_L(T)$ is not true in general. By proposition 3.5 in Kubiak [11] we know that one sufficient condition for $T = i_L \omega_L(T)$ is that L is completely distributive.

In [16] Wang extended the Lowen functor ω for completely distributive lattices as follows: For a topological space (X, T) , $(X, \omega(T))$ is called the induced space of (X, T) where $\omega(T) = \{A \in L^X : \forall \alpha \in M(L), A^{(\alpha')} \in T\}$. In 1992 Kubiak also extended the Lowen functor ω_L for a complete lattice L . In fact when L is completely distributive, $\omega_L = \omega$.

An L -topological space (X, τ) is called weakly induced space if $\forall \alpha \in M(L)$, $\forall A \in \tau$ it is true that $A^{(\alpha')} \in [\tau]$ where $[\tau]$ is the set of all crisp

open sets in τ .

Based on these facts, in this paper we use a complete, completely distributive lattice L in L^X . For a standardized basic fixed-basis terminology, we follow Hohle and Rodabaugh [10].

2. Preliminaries and Basic Definitions

2.1. Definition

[17] Let (X, τ) be an L -ts. A fuzzy point x_α is quasi coincident with $D \in L^X$ (and write $x_\alpha \prec D$) if $x_\alpha \not\leq D'$. Also D quasi coincides with E at x ($D \text{ } q \text{ } E$ at x) if $D(x) \not\leq E'(x)$. We say D quasi coincident with E and write $D \text{ } q \text{ } E$ if $D \text{ } q \text{ } E$ at x for some $x \in X$. Further $D \neg q E$ means D not quasi coincides with E . We say $U \in \tau$ is quasi coincident *nb*d of x_α (Q -*nb*d) if $x_\alpha \prec U$. The family of all Q -*nb*ds of x_α is denoted by $Q_\tau(x_\alpha)$ or $Q(x_\alpha)$.

2.2. Definition

[17] Let (X, τ) be an L -ts, $A \in L^X$. $\Phi \subset L^X$ is called a Q -cover of A if for every $x \in \text{Supp}(A)$, there exist $U \in \Phi$ such that $x_{A(x)} \prec U$. Φ is a Q -cover of (X, τ) if Φ is a Q cover of $\top$. If $\alpha \in M(L)$, then $\mathbf{C} \in \tau$ is an α - Q -*nb*d of A if $\mathbf{C} \in Q(x_\alpha)$ for every $x_\alpha \leq A$. Φ is called an α - Q -cover of A , if for each $x_\alpha \leq A$, there exists $U \in \Phi$ such that $x_\alpha \prec U$. Φ is called an open α - Q -cover of A if $\Phi \subset \tau$ and Φ is an α - Q -cover of A . $\Phi_0 \subset L^X$ is called a sub α - Q -cover of A if $\Phi_0 \subset \Phi$ and Φ_0 is also an α - Q -cover of A . Φ is called an α^- - Q cover of A , if there exists $\gamma \in \beta^*(\alpha)$ such that Φ is γ - Q -cover of A .

2.3. Definition

[17] Let (X, τ) be an L -ts, $D \in L^X$. D is called N -compact if for every $\alpha \in M(L)$, every open α - Q cover of D has a finite sub family which is an α^- - Q cover of D . (X, τ) is called N -compact if $\top$ is N -compact.

2.4. Definition

[?] Let (X, τ) be an L -ts, $\mathbf{A} = \{A_t : t \in T\} \subseteq L^X$, $x_\lambda \in M(L^X)$. $\mathbf{A}$ is called locally finite at x_λ , if there exist $U \in Q(x_\lambda)$ and a finite subset T_0 of T such that $t \in T \setminus T_0 \Rightarrow A_t \neg q U$. And $\mathbf{A}$ is called *-locally finite at x_λ if there exist $U \in Q(x_\lambda)$ and a finite subset T_0 of T such that

$t \in T_0 \Rightarrow \chi_{(A_t)_{(\perp)}} \neg q U$. $\mathbf{A}$ is called locally finite (*-locally finite) for short, if $\mathbf{A}$ is locally finite(*-locally finite) at every molecule $x_\lambda \in M(L^X)$.

2.5. Definition

[14] Let (X, τ) be an L -ts. $\mathbf{A} = \{A_t : t \in T\} \subseteq L^X$, $x_\lambda \in M(L^X)$. $\mathbf{A}$ is called point finite at x_λ if $x_\lambda \prec A_t$ for at most finitely many $t \in T$. And $\mathbf{A}$ is *-point finite at x_λ if there exists at most finitely many $t \in T$ such that $x_\lambda \prec \chi_{(A_t)_{(\perp)}}$. $\mathbf{A}$ is called point finite (resp. *-point finite) for short, if $\mathbf{A}$ is point finite (resp. *-point finite) at every molecule x_λ of L^X .

2.6. Definition

Let (X, τ) be an L -ts, $\mathbf{A} = \{A_t : t \in T\} \subseteq L^X$, $x_\lambda \in M(L^X)$. $\mathbf{A}$ is called locally countable at x_λ , if there exist $U \in Q(x_\lambda)$ and a countable subset T_0 of T such that $t \in T \setminus T_0 \Rightarrow A_t \neg q U$. And $\mathbf{A}$ is called *-locally countable at x_λ if there exist $U \in Q(x_\lambda)$ and a countable subset T_0 of T such that $t \in T_0 \Rightarrow \chi_{(A_t)_{(\perp)}} \neg q U$. $\mathbf{A}$ is called locally countable (*-locally countable) for short, if $\mathbf{A}$ is locally countable (*-locally countable) at every molecule $x_\lambda \in M(L^X)$.

The previous notions “locally countable family” is defined for L -ts. They can be also defined for L -subsets:

2.7. Definition

Let (X, τ) be an L -ts. $A \in L^X$, $\mathbf{A} = \{A_t : t \in T\} \subseteq L^X$, $x_\lambda \in M(L^X)$. $\mathbf{A}$ is called locally countable in A , if $\mathbf{A}$ is locally countable at every molecule $x_\lambda \in M(\downarrow A)$.

2.8. Definition

Let (X, τ) be an L -ts. $\mathbf{A} = \{A_t : t \in T\} \subseteq L^X$, $B \in L^X$.

$\mathbf{A}$ is called σ -locally countable in B if $\mathbf{A}$ is the countable union of sub families which are locally countable in B . $\mathbf{A}$ is called σ -locally countable for short, if $\mathbf{A}$ is σ -locally countable in $\top$.

2.9. Definition

[17] Let (X, τ) be an L -ts. Then by $[\tau]$ we denote the family of support sets of all crisp subsets in τ . $(X, [\tau])$ is a topology and it is the background

space. (X, τ) is weakly induced if $U \in \tau$ is a lower semi continuous function from the background space $(X, [\tau])$ to L .

2.10. Definition

[17] Let (X, τ) be an L -ts. (X, τ) is called weakly α -induced if $U_{(\alpha)} \in [\tau]$ for every $U \in \tau$.

2.11. Proposition

[17] Let (X, τ) be an L -ts. Then the following conditions are equivalent.

- (i) (X, τ) is weakly induced.
- (ii) (X, τ) is weakly γ -induced for every $\gamma \in pr(L)$.
- (iii) (X, τ) is weakly α -induced for every $\alpha \in L$.

2.12. Definition

[17] For a property P of ordinary topological space, a property P^* of L -ts is called a good L -extension of P , if for every ordinary topological space (X, T) , (X, T) has the property P if and only if $(X, \omega_L(T))$ has property P^* . In particular when $L = [0, 1]$ we say P^* is a good extension of P . Where $\omega_L(T)$ is the family of all lower semi continuous function from (X, T) to L .

2.13. Definition

[17] A collection $\mathbf{A}$ refines a collection $\mathbf{B}$ ($\mathbf{A} < \mathbf{B}$) if for every $A \in \mathbf{A}$, there exists $B \in \mathbf{B}$ such that $A \leq B$.

2.14. Definition

[17] Let (X, τ) be an L -ts. $\mathbf{A} = \{A_t : t \in T\} \subseteq L^X$ is a closure preserving collection if for every subfamily $\mathbf{A}_0$ of $\mathbf{A}$, $cl[\vee \mathbf{A}_0] = \vee[cl \mathbf{A}_0]$.

2.15. Proposition

[17] Let (X, τ) be an L -ts. $\mathbf{A} \subset L^X$ is closure preserving. Then for every sub family $\mathbf{A}_0 = \{A_t : t \in T\} \subset \mathbf{A}$, $\vee_{t \in T} cl A_t$ is a closed subset.

2.16. Theorem

Every locally countable family of subsets is closure preserving.

Proof. Let $\mathbf{A} \subset L^X$ is locally countable, $\mathbf{A}_0 = \{A_t : t \in T\} \subset \mathbf{A}$, then $\mathbf{A}_0$ is locally countable. Since $\vee(cl\mathbf{A}_0) \leq cl(\vee\mathbf{A}_0)$ is clear it is sufficient to prove that $cl(\vee\mathbf{A}_0) \leq \vee(cl\mathbf{A}_0)$. Suppose $x_\alpha \in M(\downarrow cl(\vee\mathbf{A}_0))$. Since $\mathbf{A}_0$ is locally countable, there exist $U \in Q(x_\alpha)$ such that $\Rightarrow A_t \neg q U$ for every $t \in T \setminus T_0$ where T_0 is a countable subset of T . This implies that $A_t \leq U'$ for every $t \in T \setminus T_0$. If $x_\alpha \not\leq \vee(cl\mathbf{A}_0)$, then $x_\alpha \not\leq cl\mathbf{A}_t$ for every $t \in T_0$ and hence there exist $U_t \in Q(x_\alpha)$ such that $A_t \leq U'_t$. Since T_0 is countable, $V = U \wedge (\vee_{t \in T_0} U_t) \in Q(x_\alpha)$ and $A_t \leq V'$ for every $t \in T$. So $\vee_{t \in T} A_t \leq V'$ and hence $x_\alpha \leq cl(\vee\mathbf{A}_0) = cl(\vee_{t \in T} A_t) \leq cl(V') = V'$. That is x_α is not quasi coincidence with V , which is a contradiction that $V \in Q(x_\alpha)$. Therefore $x_\alpha \in \vee(cl\mathbf{A}_0)$ and thus $cl(\vee\mathbf{A}_0) = \vee(cl\mathbf{A}_0)$. $\square$

2.17. Definition

[14] A collection $\mathbf{U}$ of fuzzy subsets of an L -topological space (X, τ) is said to be well monotone if the subset relation ' $<$ ' is a well order on $\mathbf{U}$.

2.18. Definition

[14] A collection $\mathbf{U}$ of fuzzy subsets of an L -topological space (X, τ) is said to be directed if $U, V \in \mathbf{U}$ implies there exists $W \in \mathbf{U}$ such that $U \vee V < W$.

2.19. Definition

Let (X, τ) be an L -ts, $A \in L^X$, $\mathbf{B} \subset L^X$. Then $st(A, \mathbf{B}) = \vee\{B \in \mathbf{B} : B q A\}$ is defined as the star of $\mathbf{B}$ about A . If $x_\lambda \in M(L^X)$, then $st(\{x_\lambda\}, \mathbf{B})$ is denoted by $st(x_\lambda, \mathbf{B})$.

2.20. Definition

Let (X, τ) be an L -ts. $\mathbf{A} = \{A_t : t \in T\} \subseteq L^X$ is a interior preserving collection if for every subfamily $\mathbf{A}_0$ of $\mathbf{A}$, $int[\wedge\mathbf{A}_0] = \wedge[int\mathbf{A}_0]$.

3. Para-Lindelof Spaces

3.1. Definition

[17] Let (X, τ) be an L -ts, $A \in L^X$, $\alpha \in M(L)$. A is called α -Lindelof if every open α - Q -cover of A has a countable subfamily which is also an α - Q -cover of A . A is Lindelof if A is α -Lindelof for every $\alpha \in M(L)$. And (X, τ) is Lindelof if $\top$ is Lindelof.

3.2. Definition

Let (X, τ) be an L -ts, $A \in L^X$, $\alpha \in M(L)$. A is called α -para-Lindelof (α^* -para-Lindelof) if for every open α - Q -cover Φ of A , there exist an open refinement Ψ of Φ which is locally countable ($*$ -locally countable) in A and Ψ is also an α - Q -cover of A . A is para-Lindelof ($*$ -para-Lindelof) if A is α -para-Lindelof (α^* -para-Lindelof) for every $\alpha \in M(L)$. (X, τ) is para-Lindelof ($*$ -para-Lindelof) if $\top$ is para-Lindelof ($*$ -para-Lindelof).

3.3. Definition

Let (X, τ) be an L -ts, $\alpha \in M(L)$. (X, τ) is called σ -para-Lindelof if for every open α - Q -cover Φ of X , there exist an open refinement Ψ of Φ which is σ -locally countable in X and also an α - Q -cover of X .

3.4. Proposition

Let (X, τ) be an L -ts, $A \in L^X$, $\alpha \in M(L)$. Then

- (i) A is α^* -para-Lindelof $\Rightarrow A$ is α -para-Lindelof.
- (ii) A is $*$ -para-Lindelof $\Rightarrow A$ is para-Lindelof.

Para-Lindelof and $*$ -Para-Lindelof are hereditary with respect to closed subsets.

3.5. Theorem

Let (X, τ) be an L -ts, $A \in L^X$, $\alpha \in M(L)$, $B \in \tau'$. Then

- (i) A is α -para-Lindelof $\Rightarrow A \wedge B$ is α -para-Lindelof.
- (ii) A is para-Lindelof $\Rightarrow A \wedge B$ is para-Lindelof.

Proof. We need to prove only (i). Suppose that $\mathbf{U}$ is an open α - Q -cover of $A \wedge B$. Take $\mathbf{V} = \mathbf{U} \cup \{B'\}$. Now clearly $\mathbf{V}$ is an open α - Q -cover of A . Since A is α -para-Lindelof, $\mathbf{V}$ has an open refinement $\mathbf{W}$ such that $\mathbf{W}$ is locally countable in A and is also an α - Q -cover of A . Take $\mathbf{W}_0 = \{W \in \mathbf{W} : \exists U \in \mathbf{U}, W \leq U\}$. Now we show that $\mathbf{W}_0$ is the required locally countable refinement of $\mathbf{V}$ which is also an α - Q -cover of $A \wedge B$. Clearly $\mathbf{W}_0$ is a locally countable refinement. Let $x_\alpha \leq A \wedge B \leq A$, since $\mathbf{W}$ is an α - Q -cover of A , there exist $W \in \mathbf{W}$ such that $x_\alpha \prec W$. Since $x_\alpha \leq B$, $B \not\leq B'$, i.e. $W \not\leq B'$. Since $\mathbf{W}$ is a refinement of $\mathbf{V} = \mathbf{U} \cup \{B'\}$, $\exists U \in \mathbf{U}$ such that $W \leq U$. Thus $W \in \mathbf{W}_0$ and hence $x_\alpha \prec W \in \mathbf{W}_0$. $\square$

A similar theorem holds for α^* -para-Lindelof and $*$ -para-Lindelof spaces also.

3.6. Theorem

Let (X, τ) be an L -ts, $A \in L^X$, $\alpha \in M(L)$, $B \in \tau'$. Then

- (i) A is α^* -para-Lindelof $\Rightarrow A \wedge B$ is α^* -para-Lindelof.
- (ii) A is * -para-Lindelof $\Rightarrow A \wedge B$ is * -para-Lindelof.

3.7. Theorem

Let (X, τ) be a weakly induced L -ts. Then the following conditions are equivalent

- (i) (X, τ) is para-Lindelof;
- (ii) There exist $\alpha \in M(L)$ such that (X, τ) is α -para-Lindelof;
- (iii) $(X, [\tau])$ is para-Lindelof.

Proof. (i) $\Rightarrow$ (ii) is obvious.

(ii) $\Rightarrow$ (iii): Let $\mathbf{U} \subset [\tau]$ be an open cover of X . Now $\mathbf{U}^* = \{\chi_U : U \in \mathbf{U}\}$ is an open α - Q -cover of $\top$ and it has a locally countable refinement $\mathbf{V}$ which is also an α - Q -cover of $\top$.

Let $\mathbf{W} = \{V_{(\alpha')} : V \in \mathbf{V}\}$. Clearly $\mathbf{W}$ is both a refinement of $\mathbf{U}$ and a cover of X . Since (X, τ) is weakly induced, we have $\mathbf{W} \subset [\tau]$. Now we want to prove that $\mathbf{W}$ is locally countable. Let $x \in X$. Since (X, τ) is α -para-Lindelof, there exist $B \in Q(x_\alpha)$ such that B only quasi coincides with a countable number of members $V_0, V_1, V_2, \dots$ of $\mathbf{V}$. Let $O = B_{(\perp)}$. By the weakly induced property of (X, τ) , $O \in [\tau]$. For every $V \in \mathbf{V}$, if $O \cap V_{(\alpha')} \neq \phi$, then there exist an ordinary point $y \in O \cap V_{(\alpha')}$, and hence $B(y) \not\leq \perp$, $V(y) \not\leq \alpha'$. Therefore $V(y)' < \alpha$ and it follows that $B(y) \not\leq V(y)'$ and thus $B \not q V$. So $V \in \{V_0, V_1, V_2, \dots\}$ and O intersects only a countable number of members $V_{0(\alpha')}, V_{1(\alpha')}, V_{2(\alpha')}, \dots$ of $\mathbf{W}$. Hence $(X, [\tau])$ is para-Lindelof.

(iii) $\Rightarrow$ (i): Suppose that $\alpha \in M(L)$ and $\mathbf{U} \subset \tau$ be an open α - Q -cover of $\top$. Since (X, τ) is weakly induced $\mathbf{U}^* = \{U_{(\alpha')} : U \in \mathbf{U}\}$ is an open cover of $(X, [\tau])$. Since $(X, [\tau])$ is para-Lindelof, there exist a refinement $\mathbf{V}$ of $\mathbf{U}^*$ which is also a locally countable cover of X . For every $V \in \mathbf{V}$, let $U_V \in \mathbf{U}$ such that $V \subset U_{V(\alpha')}$. Let $\mathbf{W} = \{\chi_V \wedge U_V : V \in \mathbf{V}\}$. Now clearly $\mathbf{W}$ is both a refinement of $\mathbf{U}$ and an α - Q -cover of $\top$. Now we will prove that $\mathbf{W}$ is locally countable. Let $x_\alpha \in M(L^X)$. Then since $\mathbf{V}$ is locally countable, there exist a neighbourhood B of x such that B intersects with V_i for countably many $V_i \in \mathbf{V}$. Now we have $\chi_B \in Q(x_\alpha)$. We will show that $\chi_B \not q \chi_{V_i} \wedge U_{V_i}$ for at most countably many i . For if possible $\chi_B \not q \chi_V \wedge U_V$ for uncountably many $V \in \mathbf{V}$. Then $\chi_B \not q \chi_V$ or χ_B

$q U_V$ for uncountably many $V \in \mathbf{V}$. In both cases B intersects with V for uncountably many $V \in \mathbf{V}$, which is a contradiction and hence W is locally countable. Therefore (X, τ) is α -para-Lindelof. This completes the proof. $\square$

3.8. Theorem

Let (X, τ) be a weakly induced L -ts. Then the following conditions are equivalent

- (i) (X, τ) is $*$ -para-Lindelof;
- (ii) There exist $\alpha \in M(L)$ such that (X, τ) is α^* -para-Lindelof;
- (iii) $(X, [\tau])$ is para-Lindelof.

Proof. (i) $\Rightarrow$ (ii) is obvious.

(ii) $\Rightarrow$ (iii): Let $\mathbf{U} \subset [\tau]$ be an open cover of X . Now $\mathbf{U}^* = \{\chi_U : U \in \mathbf{U}\}$ is an open α - Q -cover of $\top$ and it has a locally countable refinement $\mathbf{V}$ which is also an α - Q -cover of $\top$.

Take $\mathbf{W} = \{V_{(\alpha')} : V \in \mathbf{V}\}$ then $\mathbf{W}$ is both a refinement of $\mathbf{U}$ and a cover of X . Since (X, τ) is weakly induced, we have $\mathbf{W} \subset [\tau]$. Now we want to prove that $\mathbf{W}$ is locally countable. Let $x \in X$. Since (X, τ) is α^* -para-Lindelof, there exist $B \in Q(x_\alpha)$ such that $\chi_{B(\perp)}$ only quasi coincides with a countable number of members $V_0, V_1, V_2, \dots$ of $\mathbf{V}$. Then $x \in B(\perp)$. By the weakly induced property of (X, τ) , $B_{[\perp]} \in [\tau]$, so $B(\perp)$ is a neighbourhood of x . For every $V \in \mathbf{V}$, if $B(\perp) \cap V_{(\alpha')} \neq \phi$, then there exist an ordinary point $y \in B(\perp) \cap V_{(\alpha')}$, $V(y) \not\leq \alpha'$, $V(y) > \perp$, $V(y)' < \perp$. So $\chi_{B(\perp)}(y) = \top \not\leq V(y)'$, $\chi_{B(\perp)} q V$, $V \in \{V_0, V_1, V_2, \dots\}$. Therefore the neighbourhood $B(\perp)$ of x intersects a countable number of members $V_{0(\alpha')}, V_{1(\alpha')}, V_{2(\alpha')}, \dots$ of $\mathbf{W}$, thus $\mathbf{W}$ is locally countable in X . Hence $(X, [\tau])$ is para-Lindelof.

(iii) $\Rightarrow$ (i): Suppose that $\alpha \in M(L)$ and $\mathbf{U} \subset \tau$ be an open α - Q -cover of $\top$. Since (X, τ) is weakly induced $\mathbf{U}^* = \{U_{(\alpha')} : U \in \mathbf{U}\}$ is an open cover of $(X, [\tau])$. Since $(X, [\tau])$ is para-Lindelof, there exist a locally countable and open refinement $\mathbf{V}$ of $\mathbf{U}^*$ which is also a cover of X . For every $V \in \mathbf{V}$, let $U_V \in \mathbf{U}$ such that $V \subset U_{V(\alpha')}$. Let $\mathbf{W} = \{\chi_V \wedge U_V : V \in \mathbf{V}\}$. Then $\mathbf{W} \subset \tau$ is clearly a refinement of $\mathbf{U}$ and an α - Q -cover of $\top$. Now we will prove that $\mathbf{W}$ is $*$ -locally countable. Let $x_\alpha \in M(L^X)$ and $B \in Q(x_\alpha)$. If possible let $\chi(\chi_V \wedge U_V)_{(\perp)} q B$ for uncountably many $V \in \mathbf{V}$. That is $\chi_V \wedge \chi_{U_V(\perp)} q B$ for uncountably many $V \in \mathbf{V}$. And hence $\chi_V q B$ or $\chi_{U_V(\perp)} q B$ for uncountably many $V \in \mathbf{V}$. In both cases V intersects with the neighbourhood of x for uncountably many $V \in \mathbf{V}$ which is a

contradiction that $\mathbf{V}$ is locally countable. Hence W is $*$ -locally countable and this completes the proof. $\square$

3.9. Theorem

Let (X, τ) be an L -ts. Then the following are equivalent

- (i) (X, τ) is para-Lindelof;
- (ii) For every open α - Q -cover $\mathbf{A}$ of (X, τ) , there is a locally countable refinement $\mathbf{B}$ such that if $x_\alpha \in M(L^X)$ then $x_\alpha \in \text{int}(st(x_\alpha, \mathbf{B}))$.

Proof. (i) $\Rightarrow$ (ii) is obvious.

(ii) $\Rightarrow$ (i): Suppose $\mathbf{A} = \{A_t : t \in T\}$ is an open α - Q -cover of $\top$. Let $\mathbf{B} = \{B_t : t \in T\}$ be a locally countable refinement as given in (ii). Let $\mathbf{C}$ be an open α - Q -cover of $\top$ such that every element of $\mathbf{C}$ intersects at most countably many elements of $\mathbf{B}$. Then for every $x_\alpha \in M(L^X)$, there is a locally countable refinement $\mathbf{D}$ of $\mathbf{C}$ such that $x_\alpha \in \text{int}(st(x_\alpha, \mathbf{D}))$.

For each $B \in \mathbf{B}$, take $A_B \in \mathbf{A}$ such that $B \leq A_B$ and let $G_B = \text{int}(st(B, \mathbf{D})) \wedge A_B$. Then clearly $\mathbf{G} = \{G_B : B \in \mathbf{B}\}$ is an α - Q -cover of $\top$ and hence is an open refinement of $\mathbf{A}$. To show $\mathbf{G}$ is locally countable, let $x_\alpha \in M(L^X)$ and $W \in Q(x_\alpha)$ such that W intersects only countably many elements of $\mathbf{D}$. Now since each $D \in \mathbf{D}$ intersects only countably many elements of $\mathbf{B}$, it follows that W intersects only countably many elements of $\{st(B, \mathbf{D}) : B \in \mathbf{B}\}$. Hence $\mathbf{G}$ is locally countable and the theorem is proved. $\square$

Similar to Theorem 3.9 we can prove the following result:

3.10. Theorem

Let (X, τ) be an L -ts. Then the following are equivalent

- (i) (X, τ) is σ -para-Lindelof;
- (ii) For any open α - Q -cover $\mathbf{A}$ of (X, τ) , there is a σ -locally countable refinement $\mathbf{B} = \cup \mathbf{B}_i$ such that if $x_\alpha \in M(L^X)$ then $x_\alpha \in \text{int}(st(x_\alpha, \mathbf{B}_k))$ for some $k \in \mathbf{N}$.

4. Flintily Para-Lindelof Spaces

4.1. Definition

Let (X, τ) be an L -ts. $A \in L^X$, $\mathbf{A} = \{A_t : t \in T\} \subseteq L^X$, $x_\lambda \in M(L^X)$. $\mathbf{A}$ is called flintily locally countable at x_λ if there exist $U \in Q(x_\lambda) \cap \text{crs}(\tau)$

and a countable subset T_0 of T such that $t \in T \setminus T_0 \Rightarrow A_t \neg q U$. And $\mathbf{A}$ is called flintily locally countable in A , if $\mathbf{A}$ is flintily locally countable at every molecule $x_\lambda \in M(\downarrow A)$. $\mathbf{A}$ is called flintily locally countable for short, if $\mathbf{A}$ is flintily locally countable in $\top$.

4.2. Theorem

In L -ts the following implications hold

Flintily local countable $\Rightarrow$ *-local countable $\Rightarrow$ local countable

4.3. Proposition

Let (X, τ) be an L -ts, $\{A_t : t \in T\} \subseteq L^X$, $x_\lambda \in M(L^X)$. Then

(i) $\{A_t : t \in T\}$ is *-locally countable at $x_\lambda \Rightarrow \{\chi_{(A_t)_{(\perp)}} : t \in T\}$ is *-locally countable at x_λ .

(ii) $\{A_t : t \in T\}$ is flintily locally countable at $x_\lambda \Rightarrow \{\chi_{(A_t)_{(\perp)}} : t \in T\}$ is flintily locally countable at x_λ .

4.4. Theorem

Let (X, τ) be an L -ts, $A \in L^X$, $\mathbf{A} = \{A_t : t \in T\} \subseteq L^X$. If $\mathbf{A}$ is flintily locally countable in A , then $cl\mathbf{A}$ is flintily locally countable in A .

4.5. Remark

Clearly flintily local countability is strictly stronger than *-local countability. But in weakly $\perp$ -induced L -ts they are coincident with each other.

4.6. Theorem

Let (X, τ) be a weakly $\perp$ -induced L -ts, $A \in L^X$, $\mathbf{A} = \{A_t : t \in T\} \subseteq L^X$. Then $\mathbf{A}$ is flintily locally countable in A , if and only if $\mathbf{A}$ is *-locally countable in A .

Proof. By Theorem 4.2, it is enough to prove that *-local countability implies flinty local countability. Suppose $\mathbf{A}$ is *-local countable in A . Let $x_\lambda \in M(\downarrow A)$. Then there exist $U \in Q(x_\lambda)$ and a countable subset T_0 of T such that $t \in T \setminus T_0 \Rightarrow \chi_{(A_t)_{(\perp)}} \neg q U$ is satisfied. Since (X, τ) is weakly $\perp$ -induced, $U_{(\perp)} \in [\tau]$. Let $t \in T \setminus T_0$, $y \in A_{t(\perp)}$, then $U'(y) \geq \chi_{(A_t)_{(\perp)}}(y) = \top$. So $y \in U'_{[\top]} = X \setminus U_{(\perp)}$ and hence $(\chi_{U_{(\perp)}})'(y) = \top = \chi_{(A_t)_{(\perp)}}(y)$.

That is to say $\chi_{(A_t)_{(\perp)}} \leq (\chi_{U_{(\perp)}})'$, $\chi_{(A_t)_{(\perp)}} \sqcap q(\chi_{U_{(\perp)}})$. Since $\chi_{U_{(\perp)}} \in \tau$, $\chi_{U_{(\perp)}} \in Q(x_\lambda) \cap crs(\tau)$. Hence **A** is flintily locally countable. $\square$

4.7. Definition

Let (X, τ) be an L -ts, $A \in L^X$, $\alpha \in M(L)$. A is called flintily α -para-Lindelof if for every open α - Q -cover Φ of A , there exist an open refinement Ψ of Φ which is flintily locally countable in A and Ψ is also an α - Q -cover of A . A is called flintily para-Lindelof if A is flintily α -para-Lindelof for every $\alpha \in M(L)$. And (X, τ) is flintily para-Lindelof if $\top$ is flintily para-Lindelof.

By Theorem 4.2, the following implications hold:

4.8. Theorem

Let (X, τ) be an L -ts, $A \in L^X$, $\alpha \in M(L)$, then

- (i) A is flintily α -para-Lindelof $\Rightarrow A$ is α^* -para-Lindelof $\Rightarrow A$ is α -para-Lindelof.
- (ii) A is flintily para-Lindelof $\Rightarrow A$ is $*$ -para-Lindelof $\Rightarrow A$ is para-Lindelof.

Similar to Theorem 3.5 we can prove that flintily para-Lindelofness is hereditary with respect to closed subsets.

4.9. Theorem

Let (X, τ) be an L -ts, $A \in L^X$, $\alpha \in M(L)$, $B \in \tau'$. Then

- (i) A is flintily α -para-Lindelof $\Rightarrow A \wedge B$ is flintily α -para-Lindelof.
- (ii) A is flintily para-Lindelof $\Rightarrow A \wedge B$ is flintily para-Lindelof.

4.10. Theorem

In a weakly induced L -ts (X, τ) , the following are equivalent

- (i) (X, τ) is flintily para-Lindelof.
- (ii) There exist $\alpha \in M(L)$ such that (X, τ) is flintily α -para-Lindelof;
- (iii) $(X, [\tau])$ is para-Lindelof.

Proof. (i) $\Rightarrow$ (ii) is obvious.

(ii) $\Rightarrow$ (iii): Let $\mathbf{U}$ be an open cover of $(X, [\tau])$. Then $\Phi = \{\chi_U : U \in \mathbf{U}\}$ is an open α - Q -cover of $\top$ and by (ii) it has an open and flintily locally countable refinement $\Psi = \{A_t : t \in T\}$ such that Ψ is an α - Q -cover of $\top$. For every $t \in T$, take $V_t = A_{t(\alpha')}$ and $\mathbf{V} = \{V_t : t \in T\}$. Then by the weakly induced property of (X, τ) , $\mathbf{V}$ is an open cover of $(X, [\tau])$. Now we will prove $\mathbf{V}$ is a locally countable refinement of $\mathbf{U}$. Let $V_t \in \mathbf{V}$. Since

is Ψ a refinement of Φ , there exist $U \in \mathbf{U}$ such that $A_t \leq \chi_U$. Suppose $x \in V_t$, then $A_t(x) \not\leq \alpha'$, so $\chi_U(x) \neq \perp$, $x \in U$, $V_t \subset U$. Therefore $\mathbf{V}$ is a refinement of $\mathbf{U}$.

Let $x \in X$. Since Ψ is flintily locally countable, there exist $B \in Q(x_\alpha) \cap \text{crs}(\tau)$ such that $A_t q B$ for only a countable number of members A_t s in ψ . Since $B \in Q(x_\alpha)$ is crisp, $B_{(\perp)}$ is the neighbourhood of x . For every $t \in T$ if $A_t \neg q B$, then $V_t \cap B_{(\perp)} = \Phi$. So $B_{(\perp)}$ intersects with only a countable members of $\mathbf{V}$, thus $\mathbf{V}$ is locally countable. Hence $(X, [\tau])$ is paraLindelof. (iii) $\Rightarrow$ (i) suppose $\alpha \in M(L)$, $\mathbf{A} = \{A_t : t \in T\}$ is an open α - Q -cover of $\top$. For every $t \in T$ take $U_t = A_{t(\alpha')}$ and $\mathbf{U} = \{U_t : t \in T\}$. Since $\mathbf{A}$ is an open α - Q -cover of $\top$ and (X, τ) is weakly induced, $\mathbf{U}$ is an open cover of $(X, [\tau])$. Therefore by (iii), there exists an open and locally countable refinement $\mathbf{V} = \{V_s : s \in S\}$ of $\mathbf{U}$ which is also a cover of $(X, [\tau])$. For every $s \in S$ take $t(s) \in T$ such that $V_s \subset U_{t(s)}$, let $W_s = A_{t(s)} \wedge \chi_{V_s}$ then W_s is an open L -set and $W_s \leq A_{t(s)}$ for every $s \in S$. Therefore $\mathbf{W} = \{W_s : s \in S\}$ is an open refinement of $\mathbf{A}$. Now we will show that $\mathbf{W}$ is an open α - Q -cover of $\top$. Let $x_\alpha \in M(L^X)$ take $s \in S$ such that $x \in V_s$ and hence $x \in U_{t(s)}$. So $A_{t(s)}(x) \not\leq \alpha'$, $\alpha \not\leq A_{t(s)}(x)'$. Since $x \in V_s$, $\chi_{V_s} \in Q(x_\alpha)$, we have $W_s = A_{t(s)} \wedge \chi_{V_s} \in Q(x_\alpha)$. Hence $\mathbf{W}$ is an open α - Q -cover of $\top$.

Suppose $x_\alpha \in M(L^X)$, then since $\mathbf{V}$ being locally countable in $(X, [\tau])$, there exist a neighbourhood B of x in $(X, [\tau])$ such that B intersects with only countably many members of $\mathbf{V}$ say $V_{s_0}, V_{s_1}, V_{s_2}, \dots$. Then for every $s \in S \setminus \{s_0, s_1, s_2, \dots\}$, $V_s \cap B = \Phi$, $B \subset V_{s'}$ and thus $\chi_B \leq \chi_{V_{s'}} \leq A'_{t(s)} \vee \chi_{V_{s'}} = W_{s'}$. That is $\chi_B \neg q W_{s'}$. Hence $\mathbf{W}$ is flintily locally countable. This completes the proof. $\square$

5. Invariant Theorems

In this section we study the behaviour of para-Lindelof spaces under various types of fuzzy mappings.

5.1. Definition

[17] Let (X, τ) , (Y, μ) be L -topological spaces, $f : X \rightarrow Y$ be an ordinary mapping. Based on this we define the L -fuzzy mapping $f^\rightarrow : L^X \rightarrow L^Y$ and its L -fuzzy reverse mapping $f^\leftarrow : L^Y \rightarrow L^X$ by
 $f^\rightarrow : L^X \rightarrow L^Y$, $f^\rightarrow(A)(y) = \vee \{A(x) : x \in X, f(x) = y\} \forall A \in L^X, \forall y \in Y$.
 $f^\leftarrow : L^Y \rightarrow L^X$, $f^\leftarrow(B)(x) = B(f(x)), \forall B \in L^Y, \forall x \in X$.

5.2. Definition

[17] Let (X, τ) , (Y, μ) be L -topological spaces,
 $f^\rightarrow : L^X \rightarrow L^Y$ an L -fuzzy mapping. We say $f^\rightarrow$ is an L -fuzzy continuous mapping from (X, τ) to (Y, μ) if its L -fuzzy reverse mapping $f^\leftarrow : L^Y \rightarrow L^X$ maps every open subset in (Y, μ) as an open one in (X, τ) . i.e., $\forall V \in \mu$, $f^\leftarrow(V) \in \tau$.

5.3. Definition

[17] Let (X, τ) , (Y, μ) be L -topological spaces,
 $f^\rightarrow : L^X \rightarrow L^Y$ an L -fuzzy mapping. We say $f^\rightarrow$ is open if it maps every open subset in (X, τ) as an open one in (Y, μ) . i.e., $\forall U \in \tau$, $f^\rightarrow(U) \in \mu$.

5.4. Definition

[17] Let (X, τ) , (Y, μ) be L -topological spaces,
 $f^\rightarrow : L^X \rightarrow L^Y$ an L -fuzzy mapping. We say $f^\rightarrow$ is closed if it maps every closed subset in (X, τ) as a closed one in (Y, μ) . i.e., $\forall F \in \tau'$, $f^\rightarrow(F) \in \mu'$.

5.5. Definition

[1] Let (X, τ) , (Y, μ) be L -ts's, $f^\rightarrow : L^X \rightarrow L^Y$ an L -fuzzy mapping. Then $f^\rightarrow$ is perfect if it is continuous, closed and $f^\leftarrow(y)$ is N -compact for every $y \in Y$.

5.6. Result

[17] If (X, τ) , (Y, μ) are two weakly induced L -topological spaces, then
 (i) If the map $f^\rightarrow : L^X \rightarrow L^Y$ is L -fuzzy continuous, then $f : (X, [\tau]) \rightarrow (Y, [\mu])$ is continuous;
 (ii) If the map $f^\rightarrow : L^X \rightarrow L^Y$ is L -fuzzy closed, then $f : (X, [\tau]) \rightarrow (Y, [\mu])$ is closed;
 (iii) If the map $f^\rightarrow : L^X \rightarrow L^Y$ is L -fuzzy open, then $f : (X, [\tau]) \rightarrow (Y, [\mu])$ is open.

5.7. Theorem

Let (X, τ) , (Y, μ) are two weakly induced L -topological spaces. Then if $f^\rightarrow : L^X \rightarrow L^Y$ is perfect, then so is $f : (X, [\tau]) \rightarrow (Y, [\mu])$.

Proof. Let $y_\alpha \in M(L^Y)$. Since $f^\rightarrow : L^X \rightarrow L^Y$ is perfect, $f^\leftarrow(y_\alpha)$ is N -compact. Now to prove $f : (X, [\tau]) \rightarrow (Y, [\mu])$ is perfect, it is enough to prove that $f^\leftarrow(y_\alpha)$ is compact for every $y \in Y$. Now let $\mathbf{U} \in [\tau]$ be an open cover of $f^{-1}(y)$. Consider $\mathbf{U}^* = \{\chi_U : U \in \mathbf{U}\}$. This is an open α - Q -cover of $f^\leftarrow(y_\alpha)$. For, let $x_\alpha \leq f^\leftarrow(y_\alpha)$. i.e., $f^\leftarrow(y_\alpha)(x) = y_\alpha(f(x)) \geq \alpha$. Now let $U \in \mathbf{U}$ be such that $x \in U$. This is possible since U is a cover of $f^{-1}(y)$. Then $\chi_U(x) \geq y_\alpha \geq \alpha$. i.e., $\chi_U(x) \geq \alpha$ or $x_\alpha \leq \chi_U$. Hence clearly $x_\alpha q \chi_U$. Hence $\{\chi_U : U \in \mathbf{U}\}$ is an open α - Q -cover of $f^\leftarrow(y_\alpha)$. Again $f^\leftarrow(y_\alpha)$ being N -compact, there exists a finite sub collection $\mathbf{U}_s^*$ of $\mathbf{U}^*$ which is also an α^- - Q cover of $f^\leftarrow(y_\alpha)$. Let $\mathbf{U}_s^* = \{\chi_{U_1}, \chi_{U_2}, \dots, \chi_{U_k}\}$. Then clearly $\{U_1, U_2, \dots, U_k\}$ will be a finite sub cover of $f^{-1}(y)$. This completes the proof. $\square$

5.8. Theorem

$(X, \tau), (Y, \mu)$ are two weakly induced L -tss. If (X, τ) is para-Lindelof and $f^\rightarrow : L^X \rightarrow L^Y$ be a closed map with $f^\leftarrow(y_\alpha)$ Lindelof for each $y_\alpha \in M(L^Y)$, then (Y, μ) is para-Lindelof.

Proof. Let $\mathbf{U}$ be an open α - Q -cover of Y and let $\mathbf{W} = \{W_t : t \in T\}$ be a locally countable open α - Q -cover refinement of $\{f^\leftarrow(U) : U \in \mathbf{U}\}$. Now for any $y_\alpha \in M(L^Y)$, $f^\leftarrow(y_\alpha)$ is Lindelof so there is an open set G_{y_α} in L^X such that $f^\leftarrow(y_\alpha) \leq G_{y_\alpha}$ and $G_{y_\alpha} \leq W_t$ for countably many $t \in T$. Take V_{y_α} as the saturated part of G_{y_α} . Then $f^\rightarrow(V_{y_\alpha})$ is an open set about y_α . Consider $\mathbf{H} = \{f^\rightarrow(W_t) : W_t \in \mathbf{W}\}$. Now $f^\rightarrow(V_{y_\alpha})$ meeting only countably many elements of $\mathbf{H}$. Hence $\mathbf{H}$ is locally countable and it is clear that $y_\alpha \in \text{int}(st(y_\alpha, \mathbf{H}))$ for every $y_\alpha \in L^Y$. Since $\mathbf{H}$ is a refinement of $\mathbf{U}$, it follows from Theorem 3.9 that (Y, μ) is para-Lindelof. $\square$

Now by Theorem 5.7 we readily have

5.9. Theorem

$(X, \tau), (Y, \mu)$ are two weakly induced L -tss and $f^\rightarrow : L^X \rightarrow L^Y$ be a perfect map. Then (X, μ) is para-Lindelof if and only if (Y, μ) is para-Lindelof.

A similar result we can obtain for flintily para-Lindelof space also:

5.10. Theorem

$(X, \tau), (Y, \mu)$ are two weakly induced L -tss and $f^\rightarrow : L^X \rightarrow L^Y$ be a perfect map. Then (X, μ) is flintily para-Lindelof if and only if (Y, μ) is

flintily para-Lindelof.

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T. Baiju

Department of Mathematics
National Institute of Technology Calicut
Calicut 673 601
Kerala
India
e-mail : baijutmaths@gmail.com

and

Sunil Jacob John

Department of Mathematics
National Institute of Technology Calicut
Calicut 673 601
Kerala
India
e-mail : sunil@nitc.ac.in