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On binomial operator representations of certain polynomial

MUMTAZ AHMAD KHAN

and

NISAR K. S.

ALIGARH MUSLIM UNIVERSITY, INDIA

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Abstract

Based on the technique used by M. A. Khan and A. K. Shukla [4] here finite series representations of binomial partial differential operators have been used to establish operator representations of various polynomials not considered in the earlier mentioned paper. The results obtained are believed to be new.

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1. Introduction :

Recently in 2009, M. A. Khan and A. K. Shukla [4] evolved a new technique to give operator representations of certain polynomials. They give binomial and trinomial operator representation of certain polynomials. The aim of the present paper is to strengthen the technique evolved by obtaining binomial operator representations of some more polynomials not considered in the above mentioned paper.

2. The definition, notations and results used :

In deriving the operational representations of various polynomials use has been made of the fact that

$$(2.1) \quad D^\mu x^\lambda = \frac{\Gamma(1+\lambda)}{\Gamma(1+\lambda-\mu)} x^{\lambda-\mu}, \quad D \equiv \frac{d}{dx}$$

Where λ and $\mu, \lambda \geq \mu$ are arbitrary real numbers.

In particular, use has been made of the following results:

$$(2.2) \quad D^r e^{-x} = (-1)^r e^{-x}$$

$$(2.3) \quad D^r x^{-\alpha} = (\alpha)_r (-1)^r x^{-\alpha-r}, \alpha \text{ is not an integer}$$

$$(2.4) \quad D^r x^{-\alpha-n} = (\alpha+n)_r (-1)^r x^{-\alpha-n-r}$$

$$(2.5) \quad D^{n-r} x^{\alpha-1+n} = \frac{(\alpha)_n}{(\alpha)_r} x^{\alpha-1+r}$$

$$(2.6) \quad D^{n-r} x^{-\alpha} = \frac{(\alpha)_n (-1)^n}{(1-\alpha-n)_r} x^{-\alpha-n+r}, \alpha \text{ is not an integer}$$

Where n and r are denote positive integers and

$$(\alpha)_n = \alpha (\alpha + 1) \cdots (\alpha + n - 1); \quad (\alpha)_0 = 1$$

We also need the definitions of the following polynomials in terms of hypergeometric function and also their notations (see [1],[3], [9], [10]).

CESARO POLYNOMIALS

It is denoted by the symbol $g_n^{(s)}(x)$ and is defined as

$$(2.7) \quad g_n^{(s)}(x) = \binom{s+n}{n} {}_2F_1 \left[\begin{matrix} -n, & 1 \\ -s-n \end{matrix}; x \right]$$

SYLVESTER POLYNOMIALS

It is denoted by the symbol $\varphi_n(x)$ and is defined as

$$(2.8) \quad \varphi_n(x) = \frac{x^n}{n!} {}_2F_0 \left[\begin{matrix} -n, & x \\ & - \end{matrix}; x^{-1} \right]$$

SHIVELY'S PSEUDO LAGUERRE POLYNOMIALS

It is denoted by the symbol $R_n(a, x)$ and is defined as

$$(2.9) \quad R_n(a, x) = \frac{(a)_{2n}}{n!(a)_n} {}_1F_1 \left[\begin{matrix} -n \\ a+n \end{matrix}; x \right]$$

HERMITE POLYNOMIAL

It is denoted by the symbol $H_n(x)$ and is defined as

$$(2.10) \quad H_n(x) = (2x)^n {}_2F_0 \left[\begin{matrix} -\frac{1}{2}n, & -\frac{1}{2}n + \frac{1}{2} \\ & - \end{matrix}; \frac{1}{x^2} \right]$$

BEDIENT POLYNOMIALS

Bedient [2], in his study of some polynomials associated with Appell's F_2 and F_3 introduced

$$(2.11) \quad R_n(\beta, \gamma, x) = \frac{(\beta)_n (2x)^n}{n!} {}_3F_2 \left[\begin{matrix} -\frac{n}{2}, -\frac{n}{2} + \frac{1}{2}, \gamma - \beta; \\ \gamma, 1 - \beta - n; \end{matrix} \quad \frac{1}{x^2} \right]$$

$$(2.12) \quad G_n(\alpha, \beta; x) = \frac{(\alpha)_n (\beta)_n (2x)^n}{n! (\alpha + \beta)_n} {}_3F_2 \left[\begin{matrix} -\frac{1}{2}n, -\frac{1}{2}n + \frac{1}{2}, 1 - \alpha - \beta - n; \\ 1 - \alpha - n, 1 - \beta - n; \end{matrix} \quad \frac{1}{x^2} \right]$$

LOMMEL POLYNOMIAL

It is denoted by the symbol $R_{n,\nu}(\frac{1}{z})$ and is defined as

$$(2.13) \quad R_{n,\nu} \left(\frac{1}{z} \right) = (\nu)_n (2z)^n {}_2F_3 \left[\begin{matrix} -\frac{1}{2}n, -\frac{1}{2}n + \frac{1}{2}; \\ \nu, -n, 1 - \nu - n; \end{matrix} \quad -\frac{1}{z^2} \right]$$

BRAFMAN POLYNOMIAL

It is denoted by the symbol $B_n^m[\alpha_1, \alpha_2 \cdots \alpha_r, \beta_1, \beta_2 \cdots \beta_s; x]$ and is defined as

$$(2.14) \quad B_n^m[\alpha_1, \alpha_2 \cdots \alpha_r, \beta_1, \beta_2 \cdots \beta_s; x] = {}_{m+r}F_s \left[\begin{matrix} \Delta(m; -n), \alpha_1, \alpha_2, \cdots \alpha_r; \\ \beta_1, \beta_2 \cdots \beta_s; \end{matrix} \quad x \right]$$

Where $\Delta(m; n)$ stands for set of m parameters $\frac{n}{m}, \frac{n+1}{m}, \dots, \frac{n+m-1}{m}$

GOULD AND HOPPER POLYNOMIAL

It is denoted by the symbol $g_n^m(x, h)$ and is defined as

$$(2.15) \quad g_n^m(x, h) = x^n {}_mF_0 \left[\begin{matrix} \Delta(m; -n); \\ -; \end{matrix} \quad h \left(\frac{-m}{x} \right)^m \right]$$

LAHIRI POLYNOMIAL

It is denoted by the symbol $H_{n,m,\nu}(x)$ and is defined as

$$(2.16) \quad H_{n,m,\nu}(x) = (\nu x)^n {}_mF_0 \left[\begin{matrix} \Delta(m; -n); \\ -; \end{matrix} \quad - \left(\frac{-m}{\nu x} \right)^m \right]$$

MADHEKAR-THAKARE'S GENARALIZED HYPERGEOMETRIC POLYNOMIAL

It is denoted by the symbol $J_n^{(\alpha,\beta)}(x; k)$ and is defined as
(2.17)

$$J_n^{(\alpha,\beta)}(x; k) = \frac{(\alpha+1)_{kn}}{n!} {}_{k+1}F_n \left[\begin{matrix} -n, \Delta(k; \alpha + \beta + n + 1); \\ \Delta(k; \alpha + 1); \end{matrix} \left(\frac{1-x}{2} \right)^k \right]$$

SRIVASTAVA'S GENERALIZED HYPERGEOMETRIC POLYNOMIAL

It is denoted by the symbol $H_n^{(\alpha,\beta)}(x; m)$ and is defined as

$$H_n^{(\alpha,\beta)}(x; m) = \left(\begin{matrix} \alpha + (\beta + 1)n \\ n \end{matrix} \right) {}_{p+m}F_{q+m} \left[\begin{matrix} \Delta(m; -n), (a_p); \\ \Delta(m; \alpha + \beta, n + 1), (b_q); \end{matrix} x \right]$$

(2.18)

3. Operational representations :

If $D_x \equiv \frac{\partial}{\partial x}$ and $D_y \equiv \frac{\partial}{\partial y}$, M. A. Khan and A. K. Shukla [4] wrote the binomial expansion for $(D_x + D_y)^n$ as

$$(3.1) \quad (D_x + D_y)^n \equiv \sum_{r=0}^n {}^nC_r D_x^{n-r} D_y^r$$

where ${}^nC_r = \frac{n!}{r!(n-r)!}$. By writing the finite series on the right of (3.1) M.A. Khan and A.K. Shukla [4] wrote (3.1) also as

$$(3.2) \quad (D_x + D_y)^n \equiv \sum_{r=0}^n {}^nC_r D_x^r D_y^{n-r}$$

If $F(x, y)$ is a function of x and y , they obtained the following from (3.1) and (3.2)

$$(3.3) \quad (D_x + D_y)^n F(x, y) \equiv \sum_{r=0}^n \frac{(-n)_r (-1)^r}{r!} D_x^{n-r} D_y^r F(x, y)$$

(3.4)

$$(D_x + D_y)^n F(x, y) \equiv \sum_{r=0}^n \frac{(-n)_r (-1)^r}{r!} D_x^r D_y^{n-r} F(x, y)$$

In particular, if $F(x, y) = f(x)g(y)$. M.A.Khan and A.K.Shukla [4] wrote (3.3) and (3.4) in the form

(3.5)

$$(D_x + D_y)^n f(x)g(y) \equiv \sum_{r=0}^n \frac{(-n)_r (-1)^r}{r!} D_x^{n-r} f(x) D_y^r g(y)$$

(3.6)

$$(D_x + D_y)^n f(x)g(y) \equiv \sum_{r=0}^n \frac{(-n)_r (-1)^r}{r!} D_x^r f(x) D_y^{n-r} g(y)$$

Now by taking special values of $f(x)$ and $g(y)$ in (3.5), we obtain the following partial differential operator representations of the polynomials given above :

(3.7)

$$(D_x + D_y)^n \left\{ x^{-s-1} y^{-1} \right\} = (-1)^n n! x^{-s-1-n} y^{-1} g_n^{(s)} \left(\frac{x}{y} \right)$$

(3.8)

$$(1 + D_y D_z)^n \left\{ y^{-x} e^{\frac{-1}{x} z} \right\} = n! y^{-x} e^{\frac{-1}{x} z} x^{-n} \varphi_n \left(\frac{y}{x} \right)$$

(3.9)

$$(1 - D_x)^n \left\{ x^{a-1+2n} \right\} = n! x^{a-1+2n} R_n(a, x)$$

(3.10)

$$(D_u - D_x D_y)^n \left\{ u^{-1} x^{\frac{n}{2}} y^{\frac{n}{2}-\frac{1}{2}} \right\} = (-1)^n n! 2^{-n} u^{-n-1} x^{-\frac{n}{2}} y^{\frac{n}{2}-\frac{1}{2}} H_n \left(\sqrt{\frac{xy}{u}} \right)$$

$$(D_u D_v D_w + D_x D_y D_z)^n \left\{ u^{-1} v^{\gamma-1+n} w^{-\beta} x^{\frac{n}{2}} y^{\frac{n}{2}-\frac{1}{2}} z^{\beta-\gamma} \right\}$$

(3.11)

$$= (n!)^2 (\gamma)_n 2^{-n} u^{-n-1} v^{\gamma-1} w^{\beta-n} x^{-\frac{n}{2}} y^{\frac{n}{2}-\frac{1}{2}} z^{\beta-\gamma} R_n \left(\beta; \gamma; \sqrt{\frac{xyz}{uvw}} \right)$$

$$(D_u D_v D_w + D_x D_y D_z)^n \left\{ u^{-1} v^{-\alpha} w^{-\beta} x^{\frac{n}{2}} y^{\frac{n}{2}-\frac{1}{2}} z^{-1+\alpha+\beta+n} \right\}$$

(3.12)

$$= (-1)^n (n!)^2 (\alpha+\beta)_n 2^{-n} u^{-n-1} v^{-\alpha-n} w^{-\beta-n} x^{\frac{n}{2}} y^{\frac{n}{2}-\frac{1}{2}} z^{\alpha+\beta+n-1} G_n \left(\alpha; \beta; \sqrt{\frac{xyz}{uvw}} \right)$$

$$(D_u D_v D_w D_x + D_y D_z)^n \left\{ u^{-1} v^{-1} w^{\nu-1+n} x^{-\nu} y^{\frac{n}{2}} z^{\frac{n}{2}-\frac{1}{2}} \right\}$$

(3.13)

$$= (n!)^2 (\nu)_n 2^{-n} u^{-1-\frac{1}{2}} v^{-n-1} w^{\nu-1} x^{-\nu-n} y^{\frac{n}{2}} z^{-\frac{n}{2}-\frac{1}{2}} R_{n,\nu} \left(\sqrt{\frac{uvw}{yz}} \right)$$

$$(D_x D_{w_1} \cdots D_{w_s} - (-1)^{r+m} D_{y_1} \cdots D_{y_m} D_{z_1} \cdots D_{z_r})^n$$

$$\times \left\{ x^{-1} w_1^{\beta_1-1+n} \cdots w_s^{\beta_s-1+n} y_1^{\frac{n}{m}} \cdots y_m^{\frac{n-m+1}{m}} z_1^{-\alpha_1} \cdots z_r^{-\alpha_r} \right\}$$

$$= (n!) (-1)^n (\beta_1)_n \cdots (\beta_s)_n x^{-n-1} w_1^{\beta_1-1} \cdots w_s^{\beta_s-1} y_1^{\frac{n}{m}} \cdots y_m^{\frac{n-(m-n)}{m}} z_1^{-\alpha_1} \cdots z_r^{-\alpha_r}$$

$$(3.14) \quad \times B_n^m \left[\alpha_1 \cdots \alpha_s; \beta_1 \cdots \beta_s; \frac{x w_1 w_2 \cdots w_s}{y_1 y_2 \cdots y_m z_1 z_2 \cdots z_r} \right]$$

$$(D_{x^m} + (-1)^m h(m^m) D_{y_1} \cdots D_{y_m})^n \left\{ (x^{-m})^{-1} y_1^{\frac{n}{m}} \cdots y_m^{\frac{n-m+1}{m}} \right\}$$

(3.15)

$$= (-1)^n n! x^{mn+m-n} y_1^{\frac{n}{m}} \cdots y_m^{\frac{n-m+1}{m}} g_n^m \left(x y_1^{\frac{1}{m}} \cdots y_m^{\frac{1}{m}}, h \right)$$

$$\left(D_{(vx)^m} + (-1)^m (-m^m) D_{y_1} \cdots D_{y_m} \right)^n \left\{ [(vx)^{-m}]^{-1} y_1^{\frac{n}{m}} \cdots y_m^{\frac{n-m+1}{m}} \right\}$$

(3.16)

$$= (1)^n n! vx^{mn+m-n} y_1^{\frac{n}{m}} \cdots y_m^{\frac{n-m+1}{m}} H_{m,n,\nu} \left(x y_1^{\frac{1}{m}} \cdots y_m^{\frac{1}{m}}, h \right)$$

$$\left(D_{y_1} \cdots D_{y_k} - \left(\frac{-1}{2} \right)^k D_{z_1} \cdots D_{z_k} \right)^n \times \left\{ y_1^{\frac{\alpha+1}{k}-1+n} \cdots y_k^{\frac{\alpha+k}{k}-1+n} z_1^{-\frac{\alpha+\beta+n+1}{k}} \cdots z_k^{-\frac{\alpha+\beta+n+k}{k}} \right\}$$

(3.17)

$$= \frac{n!}{k^{kn}} y_1^{\frac{\alpha+1}{k}-1} \cdots y_k^{\frac{\alpha+k}{k}-1} z_1^{-\frac{\alpha+\beta+n+1}{k}} \cdots z_k^{-\frac{\alpha+\beta+n+k}{k}} J_n^{(\alpha,\beta)} \left[1 - 2 \left(\frac{y_1 y_2 \cdots y_k}{z_1 z_2 \cdots z_k} \right)^{\frac{1}{k}} ; k \right]$$

$$(D_u D_{v_1} \cdots D_{v_m} D_{w_1} \cdots D_{w_q} - (-1)^{m+r} D_{y_1} \cdots D_{y_m} D_{z_1} \cdots D_{z_r})^n$$

$$\times \left\{ u^{-1} v_1^{\frac{\alpha+\beta}{m}-1+n} \cdots v_m^{\frac{\alpha+\beta}{m}-1+m} w_1^{b_1-1+n} \cdots w_q^{b_q-1+n} y_1^{\frac{n}{m}} \cdots y_m^{\frac{n-m-1}{m}} z_1^{-a_1} \cdots z_r^{-a_r} \right\}$$

$$= (-1)^n n! \frac{(\alpha + \beta n + 1)_{mn}}{(1 + \alpha + \beta n)_m m^{mn}} (b_1)_n \cdots (b_q)_n u^{-n-1} v_1^{\frac{\alpha+\beta}{m}-1} \cdots v_m^{\frac{\alpha+\beta}{m}-1}$$

(3.18)

$$\times w_1^{b_1-1} \cdots w_q^{b_q-1} y_1^{\frac{n}{m}} \cdots y_m^{\frac{n-m-1}{m}} z_1^{-a_1} \cdots z_r^{-a_r} H_n^{(\alpha,\beta)} \left(\frac{u v_1 v_2 \cdots v_m w_1 w_2 \cdots w_q}{y_1 y_2 \cdots y_m z_1 z_2 \cdots z_r} ; m \right)$$

Some proofs are given:

Proof of (3.7):

$$\begin{aligned}
 & (D_x + D_y)^n \{x^{-s-1}y^{-1}\} \\
 &= \sum_{r=0}^n \frac{(-n)_r (-1)^r}{r!} \frac{(1+s)_n (-1)^n}{(-s-n)_r} x^{-s-1-n+r} (1)_r (-1)^r y^{-1-r} \\
 &= (-1)^n n! x^{-s-1-n} y^{-1} \begin{pmatrix} s+n \\ n \end{pmatrix} {}_2F_1 \left[\begin{matrix} -n, & 1; & x \\ -s-n; & y \end{matrix} \right] \\
 &= (-1)^n n! x^{-s-1-n} y^{-1} g_n^{(s)} \left(\frac{x}{y} \right)
 \end{aligned}$$

Proof of (3.8):

$$\begin{aligned}
 & (1 + D_y D_z)^n \left\{ y^{-x} e^{\frac{-1}{x}z} \right\} \\
 &= \sum_{r=0}^n \frac{(-n)_r (-1)^r}{r!} D_y^r \{y^{-x}\} D_z^r \left\{ e^{\frac{-1}{x}z} \right\} \\
 &= n! y^{-x} e^{\frac{-1}{x}z} x^{-n} \frac{x^n}{n!} {}_2F_0 \left[\begin{matrix} -n, & x; & -1 \\ & & xy \end{matrix} \right] \\
 &= n! y^{-x} e^{\frac{-1}{x}z} x^{-n} \varphi_n \left(\frac{y}{x} \right)
 \end{aligned}$$

Proof of (3.9):

$$\begin{aligned}
 & (1 - D_x)^n \{x^{a-1+2n}\} \\
 &= \sum_{r=0}^n \frac{(-n)_r (-1)^{n-r}}{r!} \frac{(a+n)_n}{(a+n)_r} x^{a+n-1+r} \quad \text{Using (3.6)} \\
 &= n! \frac{(a)_{2n}}{(a)_n} x^{a-1+n} {}_1F_1 \left[\begin{matrix} -n & ; & x \\ a+n; & \end{matrix} \right].
 \end{aligned}$$

$$= n!x^{a-1+n}R_n(a, x)$$

Proof of (3.10):

$$\begin{aligned} & (D_u - D_x D_y)^n \left\{ u^{-1} x^{\frac{n}{2}} y^{\frac{n}{2}-\frac{1}{2}} \right\} \\ &= \sum_{r=0}^n \frac{(-n)_r}{r!} \frac{(-1)^n n!}{(-n)_r} u^{-n-1+r} \left(\frac{-n}{2} \right)_r (-1)^r x^{\frac{n}{2}-r} \left(\frac{-n}{2} + \frac{1}{2} \right)_r (-1)^r y^{\frac{n}{2}-\frac{1}{2}-r} \\ &= (-1)^n n! u^{-n-1} x^{\frac{n}{2}} y^{\frac{n}{2}-\frac{1}{2}} {}_2F_0 \left[\begin{matrix} -\frac{n}{2}, \frac{-n}{2} + \frac{1}{2} \\ ; \end{matrix} ; \frac{u}{xy} \right] \\ &= (-1)^n n! 2^{-n} u^{-n-1} x^{-\frac{n}{2}} y^{\frac{n}{2}-\frac{1}{2}} H_n \left(\sqrt{\frac{xy}{u}} \right) \end{aligned}$$

Proof of (3.11):

$$\begin{aligned} & (D_u D_v D_w + D_x D_y D_z)^n \left\{ u^{-1} v^{\gamma-1+n} w^{-\beta} x^{\frac{n}{2}} y^{\frac{n}{2}-\frac{1}{2}} z^{\beta-\gamma} \right\} \\ &= \sum_{r=0}^n \frac{(-n)_r (-1)^r}{r!} \frac{(-1)^n n!}{(-n)_r} u^{-n-1+r} \frac{(\gamma)_n}{(\gamma)_r} v^{\gamma-1+r} \frac{(\beta)_n (-1)^n}{(1-\beta-n)_r} \\ &\times w^{-\beta-n+r} \left(\frac{-n}{2} \right)_r (-1)^r x^{\frac{n}{2}-r} \left(\frac{-n}{2} + \frac{1}{2} \right)_r (-1)^r y^{\frac{n}{2}-\frac{1}{2}-r} (\gamma-\beta)_r (-1)^r z^{\beta-\gamma-r} \\ &= n! (\beta)_n (\gamma)_n u^{-n-1} v^{\gamma-1} w^{-\beta-n} x^{\frac{n}{2}} y^{\frac{n}{2}-\frac{1}{2}} z^{\beta-\gamma} \\ &\quad \sum_{r=0}^n \frac{(\frac{-n}{2})_r (\frac{-n}{2} + \frac{1}{2})_r (\gamma-\beta)_r}{r! (\gamma)_r (1-\beta-n)_r} \left(\frac{uvw}{xyz} \right)^r \\ &= \frac{(n!)^2 (\gamma)_n}{2^n} u^{-n-1} v^{\gamma-1} w^{\beta-n} x^{-\frac{n}{2}} y^{\frac{n}{2}-\frac{1}{2}} z^{\beta-\gamma} \frac{(\beta)_n (2x)^n}{n!} {}_3F_2 \left[\begin{matrix} -\frac{n}{2}, \frac{-n}{2} + \frac{1}{2}, \gamma-\beta \\ \gamma, 1-\beta-n \end{matrix} ; \frac{uuw}{xyz} \right] \\ &= (n!)^2 (\gamma)_n 2^{-n} u^{-n-1} v^{\gamma-1} w^{\beta-n} x^{-\frac{n}{2}} y^{\frac{n}{2}-\frac{1}{2}} z^{\beta-\gamma} R_n \left(\beta; \gamma; \sqrt{\frac{xyz}{uvw}} \right) \end{aligned}$$

Proof of (3.16):

$$\begin{aligned}
 & \left(D_{(vx)^m} + (-1)^m (-m^m) D_{y_1} \cdots D_{y_m} \right)^n \left\{ [(vx)^{-m}]^{-1} y_1^{\frac{n}{m}} \cdots y_m^{\frac{n-m+1}{m}} \right\} \\
 &= \sum_{r=0}^n \frac{(-n)_r (-1)^r}{r!} \frac{(-n)_n}{(-n)_r} [(vx)^{-m}]^{-n-1+r} [(-1)^m]^r (-m)^r \\
 & \quad \times \left(\frac{-n}{m} \right)_r (-1)^r y_1^{\frac{n}{m}-r} \cdots \left(\frac{-n+m-1}{m} \right)_r (-1)^r y_m^{\frac{n-m+1}{m}-r} \\
 &= (-1)^n n! (vx)^{mn+m} y_1^{\frac{n}{m}} \cdots y_m^{\frac{n-m+1}{m}} {}_mF_0 \left[\begin{matrix} \Delta(m, n); \\ -; \end{matrix} - \left(\frac{-m}{x y_1^{\frac{1}{m}} \cdots y_m^{\frac{1}{m}}} \right)^m \right] \\
 &= (1)^n n! vx^{mn+m-n} y_1^{\frac{n}{m}} \cdots y_m^{\frac{n-m+1}{m}} H_{m,n,\nu} \left(x y_1^{\frac{1}{m}} \cdots y_m^{\frac{1}{m}}, h \right)
 \end{aligned}$$

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Mumtaz Ahmad Khan

Department of Applied Mathematics,
Faculty of Engineering and Technology,
Aligarh Muslim University,
Aligarh - 202002, U. P.,
India
e-mail: mumtaz_ahmad_khan_2008@yahoo.com

and

Nisar K. S.

Department of Applied Mathematics,
Faculty of Engineering and Technology,
Aligarh Muslim University,
Aligarh - 202002, U. P.,
India
e-mail: ksnisar1@gmail.com