Banach's and Kannan's fixed point results in fuzzy 2-metric spaces

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Abstract

In this paper we establish two common fixed point theorems in fuzzy 2- metric spaces. These theorems are generalizations of the Banach Contraction mapping principle and the Kannan's fixed point theorem respectively in fuzzy 2-metric spaces.

1. Introduction

L. A. Zadeh introduced the notion of fuzzy sets in 1965. The importance of the introduced notion of fuzzy set was realized and has successfully been applied in almost all the branches of science and technology. Recently fuzzy set theory has been applied in introducing different classes of sequences and for investigating their different properties by Tripathy and Baruah [11], Tripathy, Baruah, Et and Gungor [12], Tripathy and Borgohain [13], Tripathy and Dutta [14], Tripathy and Sarma [16]. Fuzzy topological spaces has been investigated by Tripathy and Debnath [14], Tripathy and Ray [17] and many others.

Kramosil and Michlek [10] introduced the concept of fuzzy metric space in 1975 and fixed point theorems for fuzzy metric space was first obtained by Heipern [9]. Later George and Veeramani [7] modified the notion of fuzzy metric space with the help of t-norm and defined the Hausdoff topology of fuzzy metric spaces.

Gahler ([5], [6]) investigated the properties of 2-metric space and then authors investigated contraction mapping in 2-metric spaces. The idea of fuzzy 2-metric space was used by Sharma [11] and obtained some fruitful results.

Ciric [4] has introduced a generalization of the Banach Contracion mapping principle in fuzzy metric spaces. In one theorem we have extended the result of Ciric [4] to coincidence point theorem of three mappings. In another theorem we have proved a fixed point result for generalized Kannan type mapping.

2. Preliminaries and definitions

Definition 2.1. An operation * : $[0,1] \times [0,1] \times [0,1] \to [0,1]$ is called a continuous t-norm if ([0,1], *) is an abelian topological monoid with the unit 1 such that for all $a, b, c, d, e, f \in [0,1]$

$a * b * c \leq d * e * f$ whenever $a \leq d, b \leq e, c \leq f$.

The following are some examples of t-norm.

Example 2.1. Minimum t-norm $(*_M) : *_M(x, y) = min\{x, y\}$.

Example 2.2. Product t-norm $(*_P) : *_P(x, y) = x.y$.

Example 2.3. Lukasiewicz t-norm $(*_L) : *_L(x, y) = max\{x+y-1, 0\}$.

Definition 2.2. The 3-tuple $(X, M, *)$ is called a fuzzy 2-metric space if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set in $X^3 \times [0, \infty)$ satisfying the following conditions for all $x, y, z, u \in X$ and $t_1, t_2, t_3 > 0$.

(1) $M(x, y, z, 0) = 0$.

(2) $M(x, y, z, t) = 1$ for all $t > 0$ when at least two of the three points are equal.

(3) $M(x, y, z, t) = M(x, z, y, t) = M(y, z, x, t)$ (Symmetry about three variables).

(4) $M(x, y, z, t_1+t_2+t_3) \geq M(x, y, u, t_1)*M(x, u, z, t_2)*(M(u, y, z, t_3)$. (This corresponds to tetrahedron inequality in 2-metric spaces).

(5) $M(x, y, z, .) : [0, \infty) \rightarrow [0,1]$ is left continuous.

(6) $\lim\limits_{t \to \infty} M(x, y, z, t) = 1$ for all $x, y, z \in X$ and $t > 0$.

Lemma 2.3. Let $(X, M, *)$ be a fuzzy 2-metric space. Then $M(x, y, z, .)$ is non-decreasing function for all $x, y, z \in X$.

Proof. Suppose $M(x, y, z, t) > M(x, y, z, s)$ for same $s, t > 0$ with $s > t$.

Then we have,

$M(x, y, z, t)$
$> M(x, y, z, s)$
$\geq M(x, y, y, \frac{s}{2} + \frac{t}{2}) * M(x, y, z, t) * M(y, y, z, \frac{s}{2} - \frac{t}{2})$
$\geq 1 * M(x, y, z, t) * 1$

$$= M(x, y, z, t),$$

which is a contradiction.

Hence $M(x, y, z, .)$ in non-decreasing.

Definition 2.4. A sequence $\{x_n\}$ in a fuzzy 2-metric space $(X, M, *)$ is said to converge to x in X if and only if $\lim\limits_{n \to \infty} M(x_n, x, a, t) = 1$ for all $a \in x$ and $t > 0$.

Definition 2.5. Let $(X, M, *)$ be a fuzzy 2-metric space. A sequence $\{x_n\}$ in X is called Cauchy sequence, if and only if

$$\lim\limits_{n \to \infty} M(x_{n+p}, x_n, a, t) = 1 \text{ for all } a \in X \text{ and } p > 0, t > 0.$$

Definition 2.6. A fuzzy 2-metric space $(X, M, *)$ is said to be complete if and only if every Cauchy sequence in X is convergence in X.

The following definition is given by Hadzic and Pap [7].

Definition 2.7. A t-norm * is said to be Hadzic type t-norm if the family $\{*^p\}_{p \in N}$ of its iterates defined for each $s \in (0,1)$ by $*^0(s) = 1$, $*^{(p+1)}(s) = *(*^p(s), s)$ for all $p > 0$ is equi-continuous at s=1, that is, given $\lambda > 0$, there exists $\eta(\lambda) \in (0,1)$ such that $1 \geq s > \eta(\lambda) \Rightarrow *(p)_{(s)} > 1 - \lambda$ for all $p \in N$.

Lemma 2.8. Let $(X, M, *)$ be a fuzzy 2-metric space such that $M(x, y, a, t) \to 1$ as $t \to \infty$ for all $x, y \in X$, where * is a Hadzic type t-norm. If the sequence $\{x_n\}$ in X is such that for all $n \in N$,

(2.1) $$M(x_n, x_{n+1}, a, t) \geq M(x_{n-1}, x_n, a, t/k),$$

where $0 < k < 1, t > 0$, then the sequence $\{x_n\}$ is a Cauchy sequence.

Lemma 2.9. M is a continuous function on $X^3 \times (0, \infty)$.

Definition 2.10. Two mappings $f, g : X \to X$, where X is a non-empty set, are said to be weakly compatible if they commute at their coincidence point, that is for any $x \in X$, $fx = gx$ implies that $fgx = gfx$.

Definition 2.11. (Ψ-function). A function ψ: $[0,1] \times [0,1] \to [0,1]$ is said to be a Ψ-function if

(1) ψ is monotonic increasing and continuous.

(2) $\psi(t,t) \geq t$ for all $0 \leq t \leq 1$.

Example 2.4. Some examples of Ψ-function are,

(a)$\psi(x,y) = \sqrt{xy}$.

(b)$\psi(x,y) = \min \{x,y\}$, where $x, y \in [0.1]$.

3. Main results

Theorem 3.1. Let $(X, M, *)$ be a fuzzy 2-metric space such that

$M(x,y,a,t) \to 1$ as $t \to \infty$ for all $x, y \in X$, where * is a Hadzic type t-norm and let $A, B, g : X \to X$ be three mappings such that,
(i) gX is closed.
(ii) $AX \subseteq gX$ and $BX \subseteq gX$.
(iii) M (Ax, By, a, kt) + q (1 - max (M(gx,By,a,kt), M(gy,Ax, a, kt)))
$\qquad \geq M(gx, gy, a, t)$,
(3.1)

where $x, y \in X$, $x \neq y$, $t > 0$ and $0 < k < 1$.

Then mappings A, B and g have a coincidence point.

Proof. Let $x_0 \in X$ be an arbitrary point. A sequence $\{x_n\} \in X$ is constructed by $gx_1 = Ax_0$, $gx_2 = Bx_1$, $gx_3 = Ax_2$ for all $n \in N$,

$gx_{2n-1} = Ax_{2n-2}$, $gx_{2n} = Bx_{2n-1}$.

This is possible by condition (ii) of the theorem. If $x_n = x_{n+1}$ for some n, then the proof of the theorem is trivial. So let $x_n \neq x_{n+1}$ for all $n \in N$.

Now, putting $x = x_{2n}$, and $y = x_{2n+1}$ in (3.1), for all $t > 0$,

We have,

$M(Ax_{2n}, Bx_{2n+1}, a, kt) + q(1 - maxM(gx_{2n}, Bx_{2n+1}, a, kt),$
$M(gx_{2n+1}, Ax_{2n}, a, kt))$

$\geq M(gx_{2n}, gx_{2n+1}, a, t)$

$\Rightarrow M(gx_{2n+1}, gx_{2n+2}, a, kt) + q \ (1\text{-max}M(gx_{2n}, gx_{2n+2}, a, kt),$

$M(gx_{2n+1}, gx_{2n+1}, a, kt))$
$\geq M(gx_{2n}, gx_{2n+1}, a, t)$

$\Rightarrow M(gx_{2n+1}, gx_{2n+2}, a, kt) + q(1 - 1)$

$\geq M(gx_n, gx_{2n+1}, a, t).$

Hence, for all $t > 0$; $n \geq 0$, we have

(3.2) $M(gx_{2n+1}, gx_{2n+2}, a, kt) \geq M(gx_{2n}, gx_{2n+1}, a, t).$

On considering, $x = x_{2n}$ and $y = x_{2n-1}$ in (3.1), for all $t > 0$, we have,

$M(Ax_{2n}, Bx_{2n-1}, a, kt) + q(1 - maxM(gx_{2n}, Bx_{2n-1}, a, kt),$
$M(gn_{2n-1}, Ax_{2n}, a, kt)).$

$\geq M(gx_{2n}, gx_{2n-1}, a, t).$

$\Rightarrow M(gx_{2n+1}, gx_{2n}, a, kt) + q(1 - maxM(gx_{2n}, gx_{2n}, a, kt),$

$M(gx_{2n-1}, gx_{2n+1}, a, kt)).$

$\geq M(gx_{2n}, gx_{2n-1}, a, t).$

$\Rightarrow M(gx_{2n+1}, gx_{2n}, a, kt) + q \ (1\text{-}1) \geq M(gx_{2n}, gx_{2n-1}, a, t).$

Thus, for all $t > 0, n \in N$, we have,

(3.3) $\qquad M(gx_{2n+1}, gx_{2n}, a, kt) \geq M(gx_{2n}, gx_{2n-1}, a, t).$

From, (3.2) and (3.3), for all $n \geq 0$ and $t > 0$, we have

$$M(gx_n, gx_{n+1}, a, kt) \geq M(gx_{n-1}, gx_n, a, t)$$

Then, from Lemma 2.8, we conclude that gx_n is a Cauchy sequence. Since, gX is closed, there exists $n \in X$ Such that,

(3.4) $\qquad \lim_{n \to \infty} gx_n = gx.$

From (3.4) and the fact that $x_n \neq x_{n+1}$ for all $n \in N$, without loss of generality, we assume that $x_n \neq x$ for all $n \in N$, otherwise there exists a subsequence with this property.

Putting $x = x_{2n}$ and $y = x$ in (3.1), for all $t > 0$ and $n \in N$, we have $M(Ax_{2n}, Bx, a, kt) + q(1\text{-}\max M(gx_{2n}, Bx, a, kt), M(gx, gx_{2n+1}, a, kt) \geq M(gx2n, gx, a, t).$

Taking $n \to \infty$ on both sides of the above inequality, by Lemma 2.9 and using (3.4), we have

(3.5) $\qquad \lim_{n \to \infty} M(Ax_{2n}, Bx, a, kt) = 1.$

Now, for all $t > 0$ and $n \in N$, we have

$M(gx, Bx, a, t)$

$> M(gx, gx_{2n+1}, a, t-kt)*(M(gx, Bx, gx_{2n+1}, kt/2)*M(gx_{2n+1}, Bx, a, kt/2)$

$= M(gx, gx_{2n+1}, a, t-kt)*M(gx, Bx, Ax2n, kt/2)*M(Ax_{2n}, Bx, a, kt).$

Taking $n \to \infty$ in the above inequality from (3.4) and (3.5), we have for all $t > 0$, $M(gx, Bx, a, t) = 1$, which implies that

(3.6) $\qquad\qquad\qquad Bx = gx.$

Putting $x = x$ and $y = x_{2n-1}$ in (3.1), for all $t > 0$, we have

$$M(Ax, Bx_{2n-1}, a, kt) + q(1 - maxM(gx, gx_{2n}, a, kt), M(gx_{2n-1}, Ax, a, kt))$$

$$\geq M(gx, gx_{2n-1}, a, t).$$

On taking $n \to \infty$ both sides of the above inequalities, by Lemma 2.9 and using (3.4) we have,

$$(3.7) \qquad \lim_{n \to \infty} M(Ax, Bx_{2n-1}, a, kt) = 1.$$

Now, for all $t > 0$ and $n \in N$, we have,

$$M(gx, Ax, a, t)$$

$$\geq M(gx, gx_{2n}, a, t - kt) * M(gx_{2n}, Ax, a, \tfrac{kt}{2}) * M(gx, Ax, gx_{2n}, \tfrac{kt}{2})$$

$$= M(gx, gx_{2n}, t(1-k)) * M(Bx_{2n-1}, Ax, a, \tfrac{kt}{2}) * M(gx, Ax, Bx_{2n-1}, \tfrac{kt}{2})$$

Taking $n \to \infty$ in the above inequality, using (3.4) and (3.7), we have for all $t > 0$, $M(gx, Ax, a, t) = 1$, which implies that

$$(3.8) \qquad\qquad Ax = gx.$$

From (3.6) and (3.8) we conclude that

$Ax = Bx = gx$, that is, x is a coincidence point of the mappings A, B and g.

Theorem 3.2. Let $(X, M, *)$ be a complete fuzzy 2-metric space such that $M(x, y, a, t)$ is strictly increasing in the variable t and $M(x, y, a, t) \to 1$ as $t \to \infty$ for all $x, y \in X$, where $*$ is a Hadzic type t-norm. let $A, g : X \to X$ be two self mapping on X, such that the following conditions are satisfied:

(*i*) gX is closed.

(*ii*) $AX \subseteq gX$.

(iii) $M(Ax, Ay, a, kt) + q(1\text{-}\max M(gx, Ay, a, kt), M(gy, Ax, a, kt))$

(3.9) $\geq \psi(M(gx, Ax, a, t), M(gy, Ay, a, t))$ for all $x, y \in X$,

where $q = q(x, y, a, t) \geq 0, t > 0, 0 < k < 1$ and ψ is a Ψ-function.

Then A and g have a coincidence point. Further if (A, g) is a weekly compatible pair, then A and g have an unique common fixed point.

Proof. Let $x_0 \in X$ be any point. We construct a sequence $\{x_n\}$ as follows:

$y_1 = gx_1 = Ax_0, y_2 = gx_2 = Ax_1$ and in general $y_n = gx_n = Ax_{n-1}$ for all $n \in N$.

This is possible by condition (ii) of the theorem.

Further we assume that $y_n \neq y_{n+1}$, for all $n \in N$, otherwise g and A have a coincidence point.

Thus, for all $t > 0$, $n \in N$, we have

(3.10) $0 < M(y_n, y_{n+1}, a, t) < 1.$

Putting $x = x_n$ and $y = x_{n-1}$ in (3.9) for all $t > 0$, we have

$M(Ax_n, Ax_{n-1}, a, kt) + q(1 - \max M(gx_n, Ax_{n-1}, a, kt), M(gx_{n-1}, Ax_n, a, kt))$

$\geq \psi(M(gx_n, Ax_n, a, t), M(gx_{n-1}, Ax_{n-1}, a, t)).$

$\Rightarrow M(gx_{n+1}, gx_n, a, kt) + q(1 - \max M(gx_n, gx_n, a, kt), M(gx_{n-1}, gx_{n+1}, a, kt))$

$\geq \psi(M(gx_n, gx_{n+1}, a, t), M(gx_{n-1}, gx_n, a, t)).$

$\Rightarrow M(gx_{n+1}, gx_n, a, kt) + q(1 - 1)$

$\geq \psi(M(gx_n, gx_{n+1}, a, t), M(gx_{n-1}, gx_n, a, t).$

$\Rightarrow M(y_{n+1}, y_n, a, kt) \geq (M(y_n, y_{n+1}, a, t), M(y_{n-1}, y_n, a, t).$

If $M(y_{n-1}, y_n, a, s) > M(y_n, y_{n+1}, a, s)$ for some $s > 0$, from the above inequality, using properties of y and (3.10), we obtain

$$M(y_{n+1}, y_n, a, ks) \geq \psi(M(y_n, y_{n+1}, a, s), M(y_{n+1}, y_n, a, s)$$

$$\geq (M(y_n, y_{n+1}, a, s)$$

Hence a contradiction

Thus, for all $n \in N$ and $t > 0$, we have,

$$(3.11) \qquad M(y_n, y_{n+1}, a, kt) \geq M(y_{n-1}, y_n, a, t).$$

Then from Lemma 2.8 we conclude that $\{y_n\}$ is a Cauchy sequence. Since X is complete, then there exists $z \in X$. such that $\lim\limits_{n \to \infty} y_n = z$.

Therefore,

$$(3.12) \qquad \lim_{n \to \infty} y_n = \lim_{n \to \infty} gx_n = \lim_{n \to \infty} Ax_n = z.$$

Since gX is closed, there exists $u \in X$ such that

$$(3.13) \qquad\qquad\qquad gu = z.$$

Putting $x = u$ and $y = x_{n-1}$ in (3.9), for all $t > 0$, we have

$$M(Au, Ax_{n-1}, a, kt) + q(1 - maxM(gu, Ax_{n-1}, a, kt), M(gx_{n-1}, Au, a, kt))$$

$$\geq \psi(M(gu, Au, a, t), M(gx_{n-1}, Ax_{n-1}, a, t).$$

Taking $n \to \infty$ on both sides of the above inequality, for all $t > 0$, we have,

$$M(Au, z, a, kt) + q(1 - maxM(gu, z, a, kt), M(z, Au, a, kt))$$

$$\geq \psi(M(z, Au, a, t), M(z, z, a, t).$$

$\Rightarrow M(Au, z, a, kt) + q1 - 1) \geq \psi(M(z, Au, a, t), M(z, z, a, t)).$

Thus, for all $t > 0$, using the properties of Ψ-function, we have

$M(Au, z, a, kt) \geq \psi(M(z, Au, a, t), M(z, z, a, t)$

$$\geq \psi(M(Au, z, a, t), M(Au, z, a, t)$$

$$\geq M(Au, z, a, t).$$

The above inequality implies that $Au = z$

Hence from (3.13), we have

(3.14) $$Au = gu = z.$$

Therefore, u is a coincidence point of A and g.

Next let (A, g) be a weekly compatible pair of mapping. Then from (3.14), we have $gAu = Agu$, that is

(3.15) $$gz = Az.$$

On putting $x = z, y = x_{n-1}$ in (3.9), for all $t > 0$ we have

$M(Az, z, a, kt) + q(1 - maxM(gz, z, a, kt), M(z, Az, a, kt))$

$\geq \psi(M(gz, Az, a, t), M(z, z, a, t)).$

$\Rightarrow M(Az, z, a, kt) + q(1 - maxM(Az, z, a, kt), M(z, Az, a, kt)) \geq \psi(1 - 1) = 1.$

by (3.15), that is, $M(Az, z, a, kt) + q(1 - M(Az, z, a, kt)) \geq 1.$

Thus, for all $t > 0$, $M(Az, z, a, kt) = 1.$

Which implies that

(3.16) $Az = z.$

From (3.15) and (3.16) we have

$Az = gz = z.$ So, z is a fixed point of A and g.

To prove the uniqueness, Let z_1 and z_2 be two distinct fixed point, that is,

$Az_1 = gz_1 = z_1$ and $Az_2 = gz_2 = z_2$.

Putting $x = z_1$ and $y = z_2$ in (3.9), for all $t > 0$, we have

$M(Az_1, Az_2, a, kt) + q(1 - maxM(gz_1, Az_2, a, kt), M(gz_2, Az_1, a, kt))$

$\geq \psi(M(gz_1, Az_1, a, t), M(gz_2, Az_2, a, t))$

$\geq M(z_1, z_2, a, kt) + q(1 - M(z_1, z_2, a, kt)) \geq \psi(M(z_1, z_2, a, t), M(z_2, z_2, a, t)).$

$\Rightarrow M(z_1, z_2, a, kt) + q(1 - M(z_1, z_2, a, kt)).$

$\geq \psi(1, 1) = 1.$

$\Rightarrow M(z_1, z_2, a, kt)(1 - q) \geq (1 - q).$

Therefore, for all $t > 0$, we get

$M(z_1, z_2, a, kt) = 1.$

$\Rightarrow z_1 = z_2.$

Hence the common fixed point is unique.

This establishes the Theorem.

Corollary 3.3. Let $(X, M, *)$ be a complete fuzzy 2-metric space such that $M(x, y, a, t)$ is strictly increasing in the variable t and $M(x, y, a, t) \to 1$ as $t \to \infty$ for all $x, y \in X$, where * is Hadzic type t-norm.

Let $A : X \to X$ be a self mapping on X which satisfies the following conditions for all $x, y \in X$:

$$M(Ax, Ay, a, kt) + q(1 - maxM(x, Ay, a, kt), M(y, Ax, a, kt)).$$

$$(3.17) \qquad \geq \psi(M(x, Ax, a, t)M(y, Ay, a, t)),$$

where $q = q(x, y, a, t) \geq 0, t > 0, 0 < k < 1$ and ψ is a Ψ-function. Then A has a unique fixed point.

Proof. on considering $gx = x$ for all $x \in X$ we can proof this by Theorem 3.2.

Corollary 3.4. Let $(X, M, *)$ be a complete fuzzy 2-metric space such that $M(x, y, a, t)$ is strictly increasing in the variable t and $M(x, y, a, t) \to 1$ as $t \to \infty$ for all $x, y \in X$.

Where $*$ is a Hadzic type t-norm.

Let $A : X \to X$ be self mapping on X.

Which satisfies the following inequality for all $x, y \in X$

$$(3.18) \qquad M(Ax, Ay, a, kt) \geq \psi(M(x, Ax, a, t), M(y, Ay, a, t)).$$

Where $t > 0, 0 < k < 1$ and ψ is a Ψ-function, then A has a unique fixed point.

Proof. Taking $q=0$ and $gx = x$ for all $x \in X$ in Theorem 3.2 we can proof this result.

Corollary 3.5. Let (X, d) be a complete metric space and $A : X \to X$ be a mapping which satisfies the following inequality:

$$(3.19) \qquad d(Ax, Ay) \leq \frac{k}{2}[d(x, Ax) + d(y, Ay)],$$

where $0 < k < 1$, $x, y \in X$. Then A has a unique fixed point.

Proof.   Consider the corresponding fuzzy 2-metric space $(X, M, *)$, where

$$M(x, y, a, t) = \frac{t}{t+d(x,y)} \text{ and } c * d = minc, d.$$

We prove that the inequality (3.19) implies the inequality (3.18) with

$$\psi(x, y) = min\{x, y\}.$$

Assume the contrary, then from (3.19) for some t,

$$\frac{t}{t+\frac{1}{k}d(Ax,Ay)} < min\{\frac{t}{t+d(x,Ax)}, \frac{t}{t+d(y,Ay)}\}$$

that is, $t + \frac{1}{k}d(Ax, Ay) > t + d(x, Ax)$

and $t + \frac{1}{k}d(Ax, Ay) > t + d(y, A)$

That is, $d(Ax, Ay) > \frac{k}{2}[d(x, Ax) + d(y, Ay)]$,

Hence a contradiction.

This completes the proof.

Example 3.1. Let $X = [0,1]$ and let $M(x, y, a, t) = e^{-\frac{|x-y|}{t}}$.

Let * be a Hadzic type t-norm, then $(X, M, *)$ is a complete fuzzy 2-metric space Let $A, g : X \to X$ be defined as follows:

$Ax = 1$

and
$gx = (2 + x)/3,$

where $x \in [0,1]$ and $\psi(x, y) = xy$. Then all the conditions of Theorem 3.2 are satisfied and 1 is the unique common fixed point.

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