Skolem Difference Mean Graphs

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Abstract

A graph \( G = (V, E) \) with \( p \) vertices and \( q \) edges is said to have skolem difference mean labeling if it is possible to label the vertices \( x \in V \) with distinct elements \( f(x) \) from \( 1, 2, 3, \ldots, p+q \) in such a way that for each edge \( e = uv \), let \( f^*(e) = \left| f(u) - f(v) \right| \) and the resulting labels of the edges are distinct and are from \( 1, 2, 3, \ldots, q \). A graph that admits a skolem difference mean labeling is called a skolem difference mean graph. In this paper, we prove \( C_n \circ P_m \) \( (n \geq 3, m \geq 1) \), \( T(K_{1,n_1} \circ K_{1,n_2} \circ \cdots \circ K_{1,n_m}) \), \( T(K_{1,n_1} \circ K_{1,n_2} \circ \cdots \circ K_{1,n_m}) \), \( St(n_1, n_2, \cdots, n_m) \) and \( Bt(n, n, \cdots, n) \) \( \text{m times} \) are skolem difference mean graphs.

Keywords : Mean labeling, skolem difference mean labeling, skolem difference mean graph, extra skolem difference mean labeling.

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1. Introduction

Throughout this paper by a graph we mean a finite, simple and undirected one. The vertex set and the edge set of a graph $G$ are denoted by $V(G)$ and $E(G)$ respectively. Terms and notations not defined here are used in the sense of Harary[1]. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. There are several types of labeling. An excellent survey of graph labeling is available in [2]. The concept of mean labeling was introduced by Somasundaram and Ponraj [8]. A graph $G=(V,E)$ with $p$ vertices and $q$ edges is called a mean graph if there is an injective function $f$ that maps $V(G)$ to $\{0, 1, 2, \cdots, q\}$ such that each edge $uv$ is labeled with $\frac{f(u)+f(v)}{2}$ if $f(u)+f(v)$ is even and $\frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd. Then the resulting edge labels are distinct. The notion of skolem difference mean labeling was due to Murugan and Subramanian [3]. A graph $G=(V,E)$ with $p$ vertices and $q$ edges is said to have skolem difference mean labeling if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1, 2, 3, \cdots, p+q$ in such a way that for each edge $e=uv$, let $f^*(e)=\left\lceil \frac{f(u)-f(v)}{2} \right\rceil$ and the resulting labels of the edges are distinct and are from $1, 2, 3, \cdots, q$. A graph that admits a skolem difference mean labeling is called a skolem difference mean graph. Further studies on skolem difference mean labeling are available in [4]-[7].

In this paper, we extend the study on skolem difference mean labeling and prove that $C_n \oplus P_m$ $(n \geq 3, m \geq 1)$, $T(K_{1,n_1} : K_{1,n_2} : K_{1,n_3} : \cdots : K_{1,n_m})$, $T(K_{1,n_1} \circ K_{1,n_2} \circ \cdots \circ K_{1,n_m})$, $St(n_1, n_2, \cdots, n_m)$ and $Bt(n, n, \cdots, n)$ are skolem difference mean graphs.

We use the following definitions in the subsequent section.

**Definition 1.1.** The graph $C_n \oplus P_m$ is obtained by identifying one pendant vertex of the path $P_m$ with a vertex of the cycle $C_n$.

**Definition 1.2.** The shrub $St(n_1, n_2, \cdots, n_m)$ is a graph obtained by connecting a vertex $v_0$ to the central vertex of each of $m$ number of stars.

**Definition 1.3.** The banana tree $Bt(n_1, n_2, \cdots, n_m)$ is a graph obtained by connecting a vertex $v_0$ to one leaf of each of $m$ number of stars.

**Definition 1.4.** Let $G=(V,E)$ be a skolem difference mean graph with $p$ vertices and $q$ edges. If one of the skolem difference mean labeling of $G$
satisfies the condition that all the labels of the vertices are odd, then we call this skolem difference mean labeling an extra skolem difference mean labeling and call the graph $G$ an extra skolem difference mean graph [7].

An extra skolem difference mean labeling of $P_6$ is given in Figure 1.

![Figure 1](image-url)

2. Main Results

Theorem 2.1. The graph $C_n@P_m (n \geq 3, m \geq 1)$ is a skolem difference mean graph.

Proof. Case (i): $n$ is odd.

Let $n = 2k + 1$. Let $u_1, u_2, \ldots, u_k, v_k, v_{k-1}, \ldots, v_1, v_0$ be the vertices of $C_{2k+1}$ and let $w_1, w_2, \ldots, w_m$ be the vertices of $P_m$. Then $C_n@P_m$ is obtained by identifying $w_1$ of $P_m$ with $v_0$ of $C_{2k+1}$, which has $n + m - 1$ edges. $E(C_{2k+1}@P_m) = \{v_i v_{i+1} | 1 \leq i \leq k - 1\} \cup \{u_i u_{i+1} | 1 \leq i \leq k - 1\} \cup \{w_j w_{j+1} | 1 \leq j \leq m - 1\} \cup \{v_0 v_1, v_0 u_1, u_k v_k\}$. Define $f: V(C_{2k+1}@P_m) \rightarrow \{1, 2, 3, \ldots, p + q = 2n + 2m - 2\}$ as follows:

$f(w_{2i-1}) = 2i - 1$ for $1 \leq i \leq \left\lceil \frac{m}{2} \right\rceil$,

$f(u_{2i}) = 2m + 3 - 2i$ for $1 \leq i \leq \left\lceil \frac{m}{2} \right\rceil$,

$f(u_{2i-1}) = 2m + 2n + 2 - 4i$ for $1 \leq i \leq \left\lceil \frac{k}{2} \right\rceil$,

$f(u_{2i}) = 4i$ for $1 \leq i \leq \left\lceil \frac{k}{2} \right\rceil$,

$f(v_{2i-1}) = 2m + 2n + 1 - 4i$ for $1 \leq i \leq \left\lceil \frac{k}{2} \right\rceil$,

$f(v_{2i}) = 4i + 2$ for $1 \leq i \leq \left\lceil \frac{k}{2} \right\rceil$.

For each vertex label $f$, the induced edge label $f^*$ is calculated as follows:

$f^*(w_i w_{i+1}) = m + 1 - i$ for $1 \leq i \leq m - 1$,

$f^*(u_k v_k) = 1$,

$f^*(u_i u_{i+1}) = n + m - 1 - 2i$ for $1 \leq i \leq k - 1$,
\( f^*(v_iv_{i+1}) = n + m - 2 - 2i \) for \( 1 \leq i \leq k - 1 \),
\( f^*(u_1v_0) = n + m - 1 \), \( f^*(v_1v_0) = n + m - 2 \).

**Case (ii):** \( n \) is even.

Let \( n = 2k, k > 1 \). Let \( v_0, v_1, v_2, \ldots, v_{k-1}, u_0, u_{k-1}, u_k, u_{k-2}, \ldots, u_2, u_1 \) be the vertices of \( C_{2k} \) and let \( w_1, w_2, \ldots, w_m \) be the vertices of \( P_m \). Then \( C_n @ P_m \) is obtained by identifying \( w_1 \) of \( P_m \) with \( v_0 \) of \( C_{2k} \), which has \( n + m - 1 \) edges. \( E(C_{2k}@P_m) = \{v_iv_{i+1}|1 \leq i \leq k-2\} \cup \{u_{i-1}u_i|1 \leq i \leq k-2\} \cup \{w_jw_{j+1}|1 \leq j \leq m - 1\} \cup \{v_0v_1, v_0u_1, u_0u_{k-1}, u_0v_{k-1}\} \).

**Subcase (i):** \( k > 1 \) is odd.

Define \( f : V(C_{2k}@P_m) \rightarrow \{1, 2, 3, \ldots, p + q = 2n + 2m - 2\} \) as follows:
\( f(w_{2i-1}) = 2i - 1 \) for \( 1 \leq i \leq \left\lfloor \frac{m}{2} \right\rfloor \),
\( f(w_{2i}) = 2m + 1 - 2i \) for \( 1 \leq i \leq \left\lfloor \frac{m}{2} \right\rfloor \),
\( f(w_{2i-1}) = 2m + 2n + 2 - 4i \) for \( 1 \leq i \leq \frac{k-1}{2} \),
\( f(u_{2i}) = 4i \) for \( 1 \leq i \leq \frac{k-1}{2} \),
\( f(v_{2i}) = m + 2 + 2i \) for \( 1 \leq i \leq \frac{k-1}{2} \), and \( f(u_0) = 2(n + m - k) \).

For each vertex label \( f \), the induced edge label \( f^* \) is calculated as follows:
\( f^*(w_iw_{i+1}) = m - i \) for \( 1 \leq i \leq m - 1 \),
\( f^*(w_iu_{i+1}) = n + m - 2 - 2i \) for \( 1 \leq i \leq k - 2 \),
\( f^*(v_iv_{i+1}) = n + m - 2 - 2i \) for \( 1 \leq i \leq k - 2 \),
\( f^*(v_0u_1) = n + m - 1 \),
\( f^*(v_0v_1) = n + m - 2 \),
\( f^*(u_{k-1}u_0) = m + 1 \),
\( f^*(v_{k-1}u_0) = m \).

**Subcase (ii):** \( k > 1 \) is even.

Define \( f : V(C_{2k}@P_m) \rightarrow \{1, 2, 3, \ldots, p + q = 2n + 2m - 2\} \) as follows:
\( f(w_{2i-1}) = 2i - 1 \) for \( 1 \leq i \leq \left\lfloor \frac{m}{2} \right\rfloor \),
\( f(w_{2i}) = 2m + 1 - 2i \) for \( 1 \leq i \leq \left\lfloor \frac{m}{2} \right\rfloor \),
\( f(w_{2i-1}) = 2m + 2n + 2 - 4i \) for \( 1 \leq i \leq \frac{k}{2} \),
\( f(u_{2i}) = 4i \) for \( 1 \leq i \leq \frac{k-2}{2} \),
\( f(v_{2i}) = 4i + 1 \) for \( 1 \leq i \leq \frac{k-2}{2} \),
and \( f(u_0) = 2k \).

For each vertex label \( f \), the induced edge label \( f^* \) is calculated as follows:
\( f^*(w_iw_{i+1}) = m - i \) for \( 1 \leq i \leq m - 1 \),
\( f^*(w_iu_{i+1}) = n + m - 1 - 2i \) for \( 1 \leq i \leq k - 2 \),
\( f^*(v_iv_{i+1}) = n + m - 2 - 2i \) for \( 1 \leq i \leq k - 2 \),
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\[ f^*(v_0u_1) = n + m - 1, \]
\[ f^*(v_0v_1) = n + m - 2, \]
\[ f^*(u_k-1u_0) = m + 1, \]
\[ f^*(v_k-1u_0) = m. \]

It can be verified that \( f \) is a skolem difference mean labeling. Hence \( C_n @ P_m \) is a skolem difference mean graph. \( \square \)

Skolem difference mean labelings of \( C_7 @ P_5 \) and \( C_{10} @ P_5 \) are shown in Figure 2.

\[ \text{Figure 2} \]

In the following theorem, we prove that the graph \( T(K_1,n_1 : K_1,n_2 : K_1,n_3 : \cdots : K_1,n_m) \), obtained from the stars \( K_1,n_1, K_1,n_2, \ldots, K_1,n_m \) by joining the central vertices of \( K_1,n_j \) and \( K_1,n_{j+1} \) to a new vertex \( w_j \) for \( 1 \leq j \leq m - 1 \) is an extra skolem difference mean graph.

**Theorem 2.2.** The graph \( T(K_1,n_1 : K_1,n_2 : K_1,n_3 : \cdots : K_1,n_m) \) is an extra skolem difference mean graph.

**Proof.** Let \( u_i^j (1 \leq i \leq n_j) \) be the pendant vertices and \( v_j (1 \leq j \leq m) \) be the central vertex of the star \( K_1,n_j (1 \leq j \leq m) \). Then
The graph $T(K_{1,n_1} : K_{1,n_2} : K_{1,n_3} : \cdots : K_{1,n_m})$ is a graph obtained by joining $v_j$ and $v_{j+1}$ to a new vertex $w_j (1 \leq j \leq m-1)$ by an edge. Define $f : V(T(K_{1,n_1} : K_{1,n_2} : K_{1,n_3} : \cdots : K_{1,n_m})) \to \{1, 2, 3, \cdots, p + q = 2 \sum_{k=1}^{m} n_k + 4m - 3\}$ as follows:

$$f(v_j) = 2 \sum_{k=1}^{m} n_k + 4m - 3 - 2(j - 1) \text{ for } 1 \leq j \leq m,$$

$$f(w_1) = 2n_1 + 1,$$

$$f(w_j) = 2n_1 + 1 + 2 \sum_{k=2}^{j} (n_k + 1) \text{ for } 2 \leq j \leq m - 1,$$

$$f(u^1_i) = 2i - 1 \text{ for } 1 \leq i \leq n_1$$

$$f(u^j_i) = 2 \sum_{k=1}^{j-1} (n_k + 1) + 2i - 1 \text{ for } 1 \leq i \leq n_j \text{ and } 2 \leq j \leq m.$$

For each vertex label $f$, the induced edge label $f^*$ is calculated as follows:

Let $e^j_i (1 \leq i \leq n_j \text{ and } 1 \leq j \leq m)$ be the edges joining the vertices $v_j$ with $u^j_i$. $f^*(e^j_i) = \sum_{k=1}^{m} n_k + 2m - i - 1 \text{ for } 1 \leq i \leq n_1,$

$$f^*(e^1_i) = n_j + n_{j+1} + \cdots + n_m + 2m - 2j - i + 1 \text{ for } 1 \leq i \leq n_j \text{ and } 2 \leq j \leq m,$$

$$f^*(v_1 w_1) = n_2 + n_3 + \cdots + n_m + 2(m - 1),$$

$$f^*(v_j w_j) = n_{j+1} + n_{j+2} + \cdots + n_m + 2(m - j) \text{ for } 2 \leq j \leq m - 1,$$

$$f^*(u_1 v_2) = n_2 + n_3 + \cdots + n_m + 2m - 3,$$

$$f^*(w_j v_{j+1}) = n_{j+1} + n_{j+2} + \cdots + n_m + 2(m - j) - 1 \text{ for } 2 \leq j \leq m - 1.$$

It can be verified that $T(K_{1,n_1} : K_{1,n_2} : K_{1,n_3} : \cdots : K_{1,n_m})$ is an extra skolem difference mean labeling. Hence $T(K_{1,n_1} : K_{1,n_2} : K_{1,n_3} : \cdots : K_{1,n_m})$ is an extra skolem difference mean graph. $\square$

**Corollary 2.3.** The graph $T\left(\overbrace{K_{1,n} : K_{1,n} : \cdots : K_{1,n}}^{m \text{ times}}\right)$ is an extra skolem difference mean graph.

An extra skolem difference mean labeling of $T(K_{1,7} : K_{1,7} : K_{1,7})$ is shown in Figure 3.
The graph $T(K_{1,n_1} \circ K_{1,n_2} \circ \cdots \circ K_{1,n_m})$ is obtained from the stars $K_{1,n_1}, K_{1,n_2}, \cdots, K_{1,n_m}$ by joining a leaf of $K_{1,n_j}$ and a leaf of $K_{1,n_{j+1}}$ to a new vertex $w_j (1 \leq j \leq m - 1)$ by an edge.

**Theorem 2.4.** The graph $T(K_{1,n_1} \circ K_{1,n_2} \circ K_{1,n_3} \circ \cdots \circ K_{1,n_m})$ is an extra skewes difference mean graph.

**Proof.** Let $u_{i,j}^j (1 \leq i \leq n_j)$ be the pendant vertices and $v_j (1 \leq j \leq m)$ be the central vertex of the star $K_{1,n_j} (1 \leq j \leq m)$. Then $T(K_{1,n_1} \circ K_{1,n_2} \circ K_{1,n_3} \circ \cdots \circ K_{1,n_m})$ is a graph obtained by joining $u_{i,j}^j$ and $u_{i,j}^{j+1}$ to a new vertex $w_j (1 \leq j \leq m - 1)$ by an edge. Define

$$f : V(T(K_{1,n_1} \circ K_{1,n_2} \circ K_{1,n_3} \circ \cdots \circ K_{1,n_m})) \to \{1, 2, 3, \cdots, p + q = 2 \sum_{k=1}^{m} n_k + 4m - 3\}$$

as follows:

$$f(v_j) = 2 \sum_{k=1}^{m} n_k + 4m - 4j + 1$$

for $1 \leq j \leq m$,

$$f(w_j) = 2 \sum_{k=1}^{m} n_k + 4m - 4j - 1$$

for $1 \leq j \leq m - 1$,

$$f(u_{i,j}^j) = 2i - 1$$

for $1 \leq i \leq n_1$,

$$f(u_{i,j}^{j+1}) = 2 \sum_{k=1}^{j-1} n_k + 2i - 1$$

for $1 \leq i \leq n_j$ and $2 \leq j \leq m$.

Let $e_{i,j}^j = v_j u_{i,j}^j$ for $1 \leq i \leq n_j$ and $1 \leq j \leq m$. For each vertex label $f$, the induced edge label $f^*$ is calculated as follows:

$$f^*(e_{i,j}^j) = \sum_{k=1}^{m} n_k + 2m - i - 1$$

for $1 \leq i \leq n_1$,

$$f^*(e_{i,j}^j) = n_j + n_{j+1} + \cdots + n_m + 2m - 2j - i + 1$$

for $2 \leq j \leq m - 1$ and
1 \leq i \leq n_j,
\begin{align*}
f^*(u_{i1}w_1) &= \sum_{k=1}^{m} n_k + 2m - 2 - n_1, \\
f^*(u_{ij}w_j) &= n_{j+1} + n_{j+2} + \cdots + n_m + 2(m - j) \text{ for } 2 \leq j \leq m - 1, \\
f^*(w_1u_2) &= n_2 + n_3 + \cdots + n_m + 2m - 3, \\
f^*(w_ju_{j+1}^1) &= n_{j+1} + n_{j+2} + \cdots + n_m + 2m - 2j - 1 \text{ for } 2 \leq j \leq m - 1.
\end{align*}

It can be verified that $T(K_1, n_1 \circ K_1, n_2 \circ \circ \circ K_1, n_m)$ is an extra skolem difference mean labeling. Hence $T(K_1, n_1 \circ K_1, n_2 \circ \circ \circ K_1, n_m)$ is an extra skolem difference mean graph. □

**Corollary 2.5.** The graph $T(K_1, n_1 \circ \underbrace{K_1, n \circ \circ K_1, n}_m \circ \circ \circ K_1, n)$ is a skolem difference mean graph.

A skolem difference mean labeling of $T(K_1, 6 \circ K_1, 6 \circ K_1, 6 \circ K_1, 6)$ is shown in Figure 4.
Theorem 2.6. If $G$ is a graph having $p$ vertices and $q$ edges with $q > p$ then $G$ is not a skolem difference mean graph.

Proof. Let $G$ be a $(p, q)$ graph with $q > p$. The minimum possible vertex label of $G$ is 1 and the maximum possible vertex label of $G$ is $p + q$. Therefore, the maximum possible edge label of $G$ is $\left\lceil \frac{p + q - 1}{2} \right\rceil < \left\lceil \frac{2q - 1}{2} \right\rceil = q$. That is, $G$ has no edge having the label $q$ and hence $G$ is not a skolem difference mean graph. □

Corollary 2.7. The complete graph $K_n$ is a skolem difference mean graph if and only if $n \leq 3$.

Proof. $K_2$ is $P_2$ and $K_3$ is $C_3$. The graph $K_n$ has $n$ vertices and $\frac{n(n-1)}{2}$ edges. For $n \geq 4$, $\frac{n(n-1)}{2} > n$. By theorem 2.6, $K_n$, $n \geq 4$ is not a skolem difference mean graph. □

Theorem 2.8. The shrub $St(n_1, n_2, \cdots, n_m)$ is a skolem difference mean graph.

Proof. Let $v_0, v_j, u^j_i$ ($1 \leq j \leq m, 1 \leq i \leq n_j$) be the vertices of $St(n_1, n_2, \cdots, n_m)$. Then $E(St(n_1, n_2, \cdots, n_m)) = \{v_0v_j|1 \leq j \leq m\} \cup \{v_ju^j_i|1 \leq i \leq n_j\}$.

Define $f : V(St(n_1, n_2, \cdots, n_m)) \to \left\{1, 2, 3, \cdots, p + q = 2 \sum_{j=1}^{m} n_j + 2m + 1\right\}$ as follows:

- $f(v_0) = 2m + 2 \sum_{j=1}^{m} n_j + 1$,
- $f(v_j) = 2j - 1, 1 \leq j \leq m$,
- $f(u^j_i) = 2(n_j + n_{j+1} + \cdots + n_m) + 2(j - i), 1 \leq i \leq n_j - 1$ and $1 \leq j \leq m$,
- $f(u^j_{n_j}) = 2(n_{j+1} + n_{j+2} + \cdots + n_m) + 2j + 1$ for $1 \leq j \leq m - 1$,
- $f(u^m_{n_m}) = 2m + 1$.

Let $e^j_i = v_ju^j_i$ for $1 \leq i \leq n_j$ and $1 \leq j \leq m$. For each vertex label $f$, the induced edge label $f^*$ is defined as follows: $f^*(v_0v_j) = m + n_1 + n_2 + \cdots + n_m - j + 1, 1 \leq j \leq m$

$\begin{align*}
f^*(e^j_i) & = n_j + n_{j+1} + \cdots + n_m - i + 1, 1 \leq i \leq n_j \text{ and } 1 \leq j \leq m. \\
\end{align*}$

It can be verified that $St(n_1, n_2, \cdots, n_m)$ is a skolem difference mean graph. □
The skolem difference mean labeling of $St(2, 5, 5, 3)$ is shown in Figure 5.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure5.png}
\caption{Figure 5}
\end{figure}

\textbf{Theorem 2.9.} The banana tree $Bt(n, n, \ldots, n)$ is a skolem difference mean graph.

\textbf{Proof.} Let $v_0, v_j, u_i^j$ ($1 \leq j \leq m, 1 \leq i \leq n$) be the vertices of $Bt(n, n, \ldots, n)$. Then $E(Bt(n, n, \ldots, n)) = \{v_0u_1^j, v_ju_i^j | 1 \leq i \leq n \text{ and } 1 \leq j \leq m\}$.

Define $f : V(Bt(n, n, \ldots, n)) \to \{1, 2, 3, \ldots, p + q = 2m(n + 1) + 1\}$ as follows:

\begin{align*}
f(v_0) &= 2m(n + 1) + 1, \\
f(v_j) &= 2m(n - 1) + 4j - 1, \text{ for } 1 \leq j \leq m, \\
f(u_i^1) &= 2j - 1, 1 \leq j \leq m, \\
f(u_i^j) &= (2i - 2)m + 2j - 1, \text{ for } 2 \leq i \leq n - 1 \text{ and } 1 \leq j \leq m, \\
f(u_n^j) &= 2m(n + 1) - 2(m - j) \text{ for } 1 \leq j \leq m. \text{ Let } e_i^j = v_ju_i^j \text{ for } 1 \leq i \leq n \text{ and } 1 \leq j \leq m. \text{ The induced edge label } f^* \text{ is calculated as follows: } f^*(v_0u_1^j) = m(n + 1) - j + 1, \text{ for } 1 \leq j \leq m \\
f^*(e_i^j) &= m(n - i) + j, \text{ for } 1 \leq i \leq n - 1 \text{ and } 1 \leq j \leq m, \\
f^*(v_ju_n^j) &= m - j + 1 \text{ for } 1 \leq j \leq m.
\end{align*}
It can be verified that $B_t(n, n, \ldots, n)$ is a skolem difference mean graph.

\[ n \text{ times} \]

A skolem difference mean labeling of $B_t(3,3,3,3,3,3,3,3)$ is shown in Figure 6.

![Figure 6](image-url)

References


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