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# Odd Vertex equitable even labeling of cyclic snake related graphs

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**Abstract**

*Let $G$ be a graph with $p$ vertices and $q$ edges and $A = \{1, 3, ..., q\}$ if $q$ is odd or $A = \{1, 3, ..., q + 1\}$ if $q$ is even. A graph $G$ is said to admit an odd vertex equitable even labeling if there exists a vertex labeling $f : V(G) \to A$ that induces an edge labeling $f^*$ defined by $f^*(uv) = f(u) + f(v)$ for all edges $uv$ such that for all $a$ and $b$ in $A$, $|v_f(a) - v_f(b)| \leq 1$ and the induced edge labels are $2, 4, ..., 2q$ where $v_f(a)$ be the number of vertices $v$ with $f(v) = a$ for $a \in A$. A graph that admits an odd vertex equitable even labeling is called an odd vertex equitable even graph. Here, we prove that the graph $nC_4$-snake, $CS(n_1, n_2, ..., n_k)$, $n_i \equiv 0(mod4)$,$n_i \geq 4$, be a generalized $kC_n$ -snake, $T\widehat{O}QS_n$ and $T\widetilde{O}QS_n$ are odd vertex equitable even graphs.*

**Keywords :** *vertex equitable labeling, vertex equitable graph, odd vertex equitable even labeling, odd vertex equitable even graph.*

**AMS Subject Classification (2010) :** *05C78*

## 1. Introduction

All graphs considered here are simple, finite, connected and undirected. Let $G(V, E)$ be a graph with $p$ vertices and $q$ edges. We follow the basic notations and terminology of graph theory as in [3]. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions and a detailed survery of graph labeling can be found in [2]. The vertex set and the edge set of a graph are denoted by $V(G)$ and $E(G)$ respectively.

The concept of vertex equitable labeling was due to Lourdusamy and Seenivasan in [16] and further studied in [5]-[14]. Let $G$ be a graph with $p$ vertices and $q$ edges and $A = \{0, 1, 2, ..., \lceil \frac{q}{2} \rceil\}$. A graph $G$ is said to be vertex equitable if there exists a vertex labeling $f : V(G) \rightarrow A$ that induces an edge labeling $f^*$ defined by $f^*(uv) = f(u) + f(v)$ for all edges $uv$ such that for all $a$ and $b$ in $A$, $|v_f(a) - v_f(b)| \leq 1$ and the induced edge labels are $1, 2, 3, ..., q$, where $v_f(a)$ is the number of vertices $v$ with $f(v) = a$ for $a \in A$. The vertex labeling $f$ is known as vertex equitable labeling. A graph $G$ is said to be a vertex equitable if it admits a vertex equitable labeling.

Motivated by the concept of vertex equitable labeling of graphs, Jeyanthi, Maheswari and Vijaya Lakshmi defined a new labeling namely *odd vertex equitable even labeling* [15]. A graph $G$ with $p$ vertices and $q$ edges and $A = \{1, 3, ..., q\}$ if $q$ is odd or $A = \{1, 3, ..., q + 1\}$ if $q$ is even. A graph $G$ is said to admit an odd vertex equitable even labeling if there exists a vertex labeling $f : V(G) \rightarrow A$ that induces an edge labeling $f^*$ defined by $f^*(uv) = f(u) + f(v)$ for all edges $uv$ such that for all $a$ and $b$ in $A$, $|v_f(a) - v_f(b)| \leq 1$ and the induced edge labels are $2, 4, ..., 2q$ where $v_f(a)$ be the number of vertices $v$ with $f(v) = a$ for $a \in A$. A graph that admits an odd vertex equitable even labeling is called an odd vertex equitable even graph. In [15] they proved that the graphs like path, $P_n \odot P_m (n, m \geq 1)$, $K_{1,n} \bigcup K_{1,n-2} (n \geq 3)$, $K_{2,n}$, $T_p$ -tree, a ladder $L_n$, arbitrary super subdivision of any path $P_n$ are odd vertex equitable even graphs.

Also they proved that the graphs $K_{1,n}$ is an odd vertex equitable even graph iff $n \leq 2$, the graph $G = K_{1,n} \bigcup K_{1,n-2} (n \geq 3)$ is an odd vertex equitable even graph and cycle $C_n$ is an odd vertex equitable even graph if $n \equiv 0$ or $1$ $(mod 4)$. In addition, they proved that if every edge of a graph $G$ is an edge of a triangle, then $G$ is not an odd vertex equitable even graph.

We use the following definitions in the subsequent section.

**Theorem 1.1.** *The cycle* $C_n$ *is an odd vertex equitable even graph if* $n \equiv 0$ *or 1* $(mod 4)$.

**Theorem 1.2.** *Let* $G_1(p_1, q_1)$, $G_2(P_2, q_2)$,...,$G_m(p_m, q_m)$ *be an odd vertex equitable even graphs with* $\sum_{i=1}^{m-1} q_i$ *is even,* $q_m$ *is even or odd and* $u_i, v_i$ *be the vertices of* $G_i(1 \leq i \leq m)$ *labeled by 1,* $q_i$ *if* $q_i$ *is odd or* $q_i + 1$ *if* $q_i$ *is even. Then the graph* $G$ *obtained by identifying* $v_1$ *with* $u_2$ *and* $v_2$ *with* $u_3$ *and* $v_3$ *with* $u_4$ *and so on until we identify* $v_{m-1}$ *with* $u_m$ *is also an odd vertex equitable even graph.*

**Definition 1.3.** *The corona* $G_1 \odot G_2$ *of the graphs* $G_1$ *and* $G_2$ *is defined as a graph obtained by taking one copy of* $G_1$ *(with p vertices) and p copies of* $G_2$ *and then joining the* $i^{th}$ *vertex of* $G_1$ *to every vertex of the* $i^{th}$ *copy of* $G_2$.

**Definition 1.4.** *Let* $G_1$ *be a graph with* $p$ *vertices and* $G_2$ *be any graph. A graph* $G_1 \hat{o} G_2$ *is obtained from* $G_1$ *and* $p$ *copies of* $G_2$ *by identifying one vertex of* $i^{th}$ *copy of* $G_2$ *with* $i^{th}$ *vertex of* $G_1$.

**Definition 1.5.** [1] *A* $kC_n$ *-snake is defined as a connected graph in which all the k-blocks are isomorphic to the cycle* $C_n$ *and the block-cut point graph is a path. Let* $P$ *be the path of minimum length that contains all the cut vertices of a* $kC_n$ *-snake. Barrientos proved that any* $kC_n$ *-snake is represented by a string* $s_1, s_2, ..., s_{k-2}$ *of integers of length* $k - 2$ *where the* $i^{th}$ *integer,* $s_i$ *on the string is the distance between* $i^{th}$ and $(i+1)^{th}$ *cut vertices on the path* $P$ *from one extreme and is taken from* $S_n = \{1, 2, ..., \lfloor \frac{n}{2} \rfloor\}$. *The strings obtained for both extremes are assumed to be the same. Then there are at most* $\lfloor \frac{n}{2} \rfloor^{k-2}$ *non isomorphic* $kC_n$ *-snakes. For example, the string of a* $10C_4$ *-snake is shown in Figure 1.1 is 2,2,1,2,1,1,2,1. A* $kC_n$ *-snake is said to be linear if each integer of its string is* $\lfloor \frac{n}{2} \rfloor$.

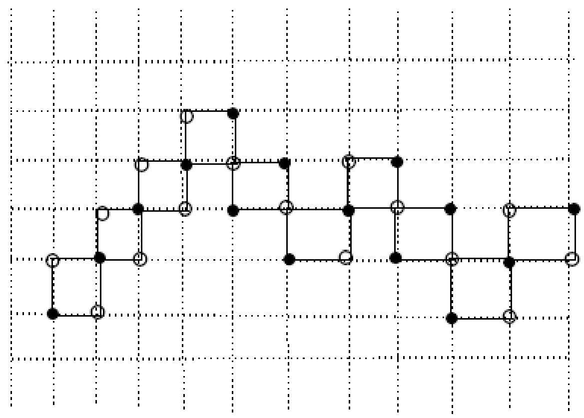

*Figure 1.1: An embedding of $10C_4$-snake*

*A $nC_k$ -snake is said to be linear if each integer of its string is $\left\lfloor \frac{k}{2} \right\rfloor$. The linear $nC_4$ -snake graph with diagonal vertices $u_{1j}$ $(1 \leq j \leq n+1)$, left to the diagonal vertices $v_{1j}$ $(1 \leq j \leq n)$ and right to the diagonal vertices $w_{1j}$ $(1 \leq j \leq n)$ is denoted by $QS_n$. For example, a linear $3C_4$ -snake graph $QS_3$ is shown in Figure 1.2.*

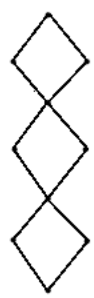

*Figure 1.2: A linear $3C_4$-snake $QS_3$*

**Definition 1.6.** *A generalized $kC_n$ -snake is defined as a connected graph in which each block is isomorphic to a cycle $C_n$ for some $n$ and the block-cut point graph is a path. It is denoted by $CS(n_1, n_2, ..., n_k)$ where $B_1$,$B_2$,...,$B_k$ are the consecutive blocks and $B_i$ is isomorphic to $C_{n_i}$. By applying the same methods used to obtain the strings of a $kC_n$ -snake, we can show that any generalized $kC_n$ -snake can also be represented by a string of integers $s_1$,$s_2$,...,$s_{k-2}$ of length $k-2$ where $s_{i-1} \in S_{n_i}$.*

**Definition 1.7.** [4] *Let $T$ be a tree and $u_0$ and $v_0$ be the two adjacent vertices in $T$. Let $u$ and $v$ be the two pendant vertices of $T$ such that the length of the path $u_0 - u$ is equal to the length of the path $v_0 - v$. If the edge $u_0v_0$ is deleted from $T$ and $u$ and $v$ are joined by an edge $uv$, then such a transformation of $T$ is called an elementary parallel transormation (or an ept) and the edge $u_0v_0$ is called transformable edge. If by the sequence of ept's, $T$ can be reduced to a path, then $T$ is called a $T_p$ -tree (transformed tree) and such sequence regarded as a composition of mappings (ept's) denoted by $P$, is called a parallel transformation of $T$. The path, the image of $T$ under $P$ is denoted as $P(T)$. A $T_p$ -tree and the sequence of two ept's reducing it to a path are illustrated in Figure 1.3.*

*(a) A $T_p$ -tree $T$* *(b) An ept $P_1(T)$* *(c) Second ept $P_2(T)$*

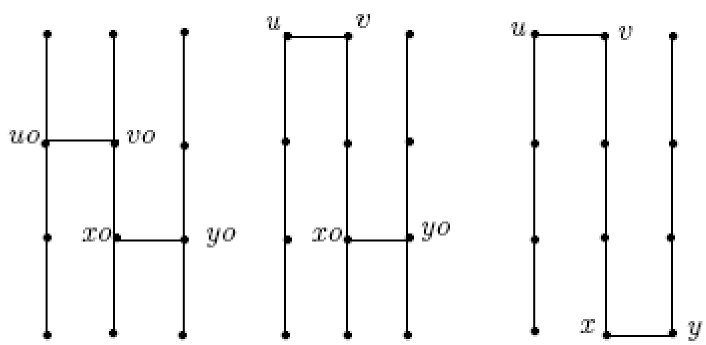


*Figure 1.3*

## 2. Main Results

*In this section, we prove that $nC_4$-snake, $CS(n_1, n_2, ..., n_k)$, $n_i \equiv 0(mod4)$, $n_i \geq 4$, be a generalized $kC_n$ -snake, $T\widehat{O}QS_n$ and $T\widetilde{O}QS_n$ are odd vertex equitable even graphs.*

**Theorem 2.1.** *The $nC_4$ -snake is an odd vertex equitable even graph.*

**Proof.** *Let $G$ be a $nC_4$ -snake with $n$ blocks and $G_i = C_4, 1 \leq i \leq n-1$ and $u_i, v_i$ be the vertices with labels 1 and $q+1$ respectively. By Theorem 1.2, $nC_4$ admits an odd vertex equitable even labeling. An example for odd vertex equitable even labeling of $3C_4$ -snake is shown in Figure 2.1.*

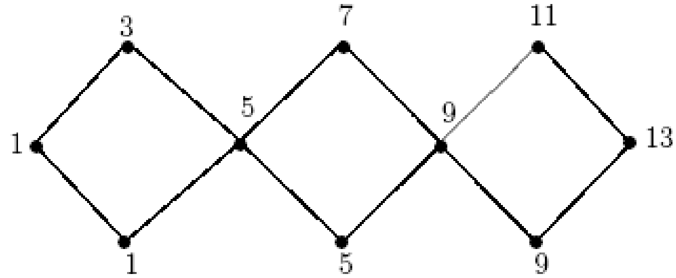


Figure 2.1

**Theorem 2.2.** *Let* $G = CS(n_1, n_2, ..., n_k)$,$n_i \equiv 0(mod4)$, $n_i \geq 4$ *be a generalized* $kC_n$ *-snake with its strings* $s_1$,$s_2$,...,$s_{k-2}$ *where* $s_i \in \{1\}, 1 \leq i \leq k$. *Then* $G$ *is an odd vertex equitable even graph.*

**Proof.** *By Theorem 1.1, the cycle* $C_n$ *is an odd vertex equitable even graph if* $n \equiv 0(mod4)$. *By Theorem 1.2,* $CS(n_1, n_2, ..., n_k)$, $n_i \equiv 0(mod4)$, *is an odd vertex equitable even graph. An example for odd vertex equitable even labeling of* $CS(8, 4, 12)$ *is shown in Figure 2.2.*

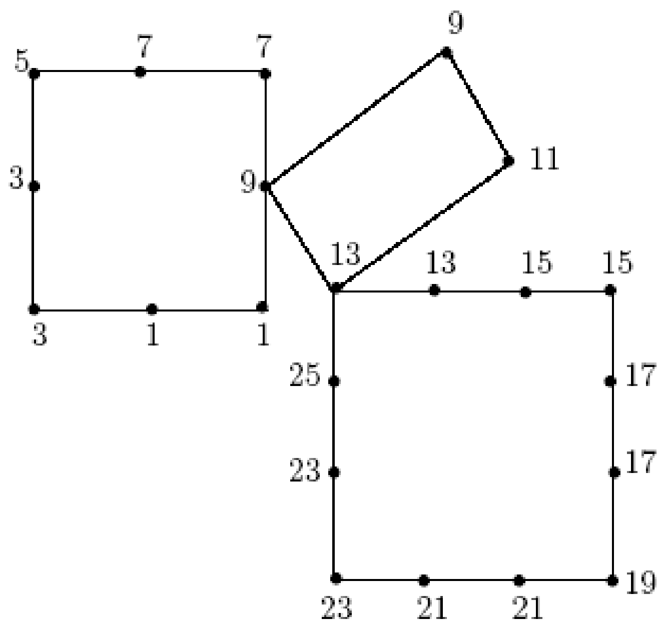


Figure 2.2

**Theorem 2.3.** *If $T$ be a $T_p$ -tree on $m$ vertices, then the graph $T\widehat{O}QS_n$ is an odd vertex equitable even graph.*

**Proof.** *Let $T$ be a $T_p$ -tree with $m$ vertices. By the definition of a transformed tree there exists a parallel transformation $P$ of $T$ such that for the path $P(T)$ we have (i) $V(P(T)) = V(T)$ (ii) $E(P(T)) = (E(T) - E_d) \bigcup E_p$ where $E_d$ is the set of edges deleted from $T$ and $E_p$ is the set of edges newly added through the sequence $P = (P_1, P_2, ..., P_k)$ of the epts $P$ used to arrive at the path $P(T)$. Clearly, $E_d$ and $E_p$ have the same number of edges.*

*Now denote the vertices of $P(T)$ successively by $u_1^{'}, u_2^{'}, ..., u_m^{'}$ starting from one pendant vertex of $P(T)$ right up to the other one.*
*Let $u_{i1}, u_{i2}, ..., u_{i(n+1)}, v_{i1}, v_{i2}, ..., v_{in}$ and $w_{i1}, w_{i2}, ..., w_{in} (1 \leq i \leq m)$ be the vertices of $i^{th}$ copy of $P_n$ with $u_{i(n+1)} = u_i^{'}$.*

*Then $V(T\widehat{O}QS_n) = \{u_{ij} : 1 \leq i \leq m, 1 \leq j \leq n+1$ with $u_{i(n+1)} = u_i^{'}\} \bigcup \{u_i^{'}, v_{ij}, w_{ij} : 1 \leq i \leq m, 1 \leq j \leq n\}$ and*
*$E(T\widehat{O}QS_n) = \{e_i^{'} = u_i^{'}u_{i+1}^{'} : 1 \leq i \leq m-1\} \bigcup E(QS_n)$.*
*Here $\left|V(T\widehat{O}QS_n)\right| = m(3n+1)$ and $\left|E(T\widehat{O}QS_n)\right| = 4mn + m - 1$.*
*Let $A = \{1, 3, ..., 4mn + m - 1\}$.*

*Define a vertex labeling $f : V(T\widehat{O}QS_n) \to A$ as follows:*

*For $1 \leq i \leq m, 1 \leq j \leq n+1$* $f(u_{ij}) = \begin{matrix} (4n+1)(i-1) + 4j - 3 & \text{if } i \text{ is odd} \\ (4n+1)i - (4j-3) & \text{if } i \text{ is even} \end{matrix}$.

*For $1 \leq i \leq m, 1 \leq j \leq n$.*

$$f(v_{ij}) = f(u_{ij}),\ f(w_{ij}) = \begin{matrix} (4n+1)(i-1) + 4j - 1 & \text{if } i \text{ is odd} \\ (4n+1)i - (4j-1) & \text{if } i \text{ is even} \end{matrix}.$$

*For the vertex labeling $f$, the induced edge labeling $f^*$ is as follows:*

*For $1 \leq i \leq m-1$ $f^*(e_i^{'}) = 2(4n+1)i$.*

*The induced edge labels of $QS_n$ are $2(4n+1)(i-1)+2j(1 \leq i \leq m, 1 \leq j \leq 2n)$ if $i$ is odd and $2(4n+1)(i-1)+2j(1 \leq i \leq m, 1 \leq j \leq 2n)$ if $i$ is even.*

*Let $v_iv_j$ be a transformed edge in $T$ for some indices $i, j, 1 \leq i \leq j \leq m$.*

*Let $P_1$ be the ept that deletes the edge $v_iv_j$ and adds an edge $v_{i+t}v_{j-t}$ where $t$ is the distance of $v_i$ from $v_{i+t}$ and the distance of $v_j$ from $v_{j-t}$.*

*Let $P$ be a parallel transformation of $T$ that contains $P_1$ as one of the constituent epts. Since $v_{i+t}v_{j-t}$ is an edge in the path $P(T)$, it follows that $i+t+1 = j-t$ which implies $j = i+2t+1$.*

*Therefore, $i$ and $j$ are of opposite parity, that is, $i$ is odd and $j$ is even or vice-versa.*

*The induced label of the edge $v_iv_j$ is given by $f^*(v_iv_j) = f^*(v_iv_{i+2t+1}) = f(v_i) + f(v_{i+2t+1}) = 2(4n+1)(i+t)$ and*

$$f^*(v_{i+t}v_{j-t}) = f^*(v_{i+t}v_{i+t+1}) = f(v_{i+t}) + f(v_iv_{i+t+1}) = 2(4n+1)(i+t).$$

*Therefore, $f^*(v_iv_j) = f^*(v_{i+t}v_{j-t})$. It can be verified that the induced edge labels of $T\widehat{O}QS_n$ are $2, 4, 6, ..., 8mn+2m-2$ and $|v_f(a) - v_f(b)| \le 1$ for all $a, b \in A$.*
*Hence, $T\widehat{O}QS_n$ is an odd vertex equitable even graph.*
*An example for odd vertex equitable even labeling of $T\widehat{O}QS_2$ where $T$ is a $T_p$ -tree on 8 vertices is shown in Figure 2.3.*

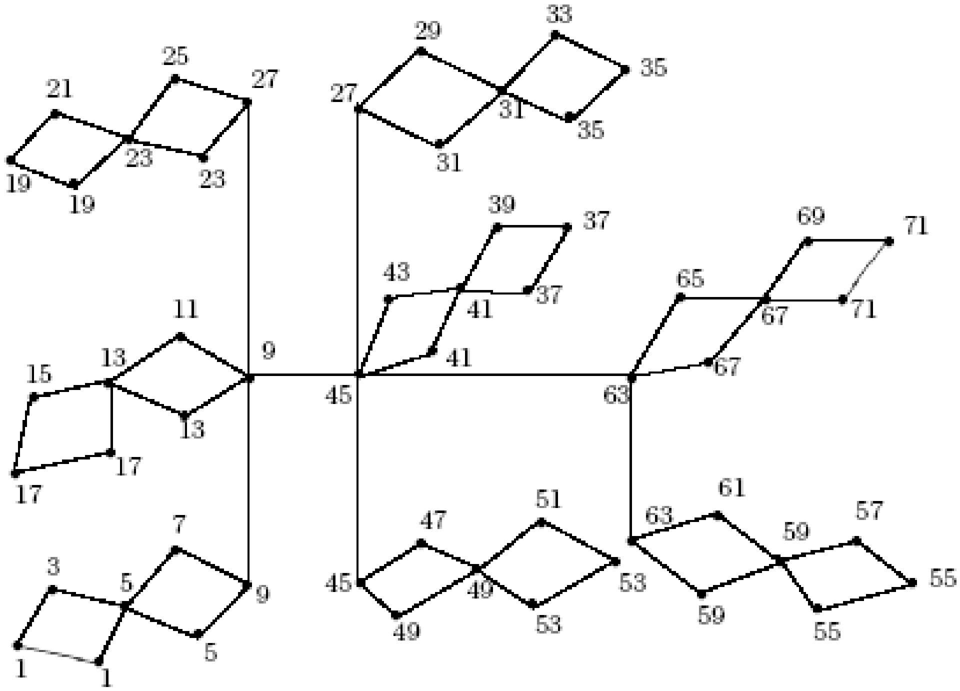


Figure 2.3

□

**Theorem 2.4.** *Let $T$ be a $T_p$ -trees on $m$ vertices. Then the graph $T\widetilde{O}QS_n$ is an odd vertex equitable even graph.*

**Proof.** *Let $T$ be a $T_p$ -tree with $m$ vertices. By the definition of a transformed tree there exists a parallel transformation $P$ of $T$ such that for the path $P(T)$ we have (i) $V(P(T)) = V(T)$ (ii) $E(P(T)) = (E(T) - E_d) \bigcup E_p$ where $E_d$ is the set of edges deleted from $T$ and $E_p$ is the set of edges newly added through the sequence $P = (P_1, P_2, ..., P_k)$ of the epts $P$ used to arrive at the path $P(T)$. Clearly, $E_d$ and $E_p$ have the same number of edges. Now denote the vertices of $P(T)$ successively by $u_1^{'}, u_2^{'}, ..., u_m^{'}$ starting from one pendant vertex of $P(T)$ right up to the other one.*
*Let $u_{i1}, u_{i2}, ..., u_{i(n+1)}, v_{i1}, v_{i2}, ..., v_{in}$ and $w_{i1}, w_{i2}, ..., w_{in} (1 \leq i \leq m)$ be the vertices of $i^{th}$ copy of $P_n$.*

Then $V(T\widetilde{O}QS_n) = \{u_{ij} : 1 \leq i \leq m, 1 \leq j \leq n+1\} \bigcup \{u_i^{'}, v_{ij}, w_{ij} : 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(T\widetilde{O}QS_n) = E(QS_n) \bigcup \{e_i^{'} = u_i^{'}u_{i+1}^{'} : 1 \leq i \leq m-1\} \bigcup \{e_i^{''} = u_i^{'}u_{i(n+1)} : 1 \leq i \leq m\}$.
Here $\left|V(T\widetilde{O}QS_n)\right| = m(3n+2)$ and $\left|E(T\widetilde{O}QS_n)\right| = 4mn + 2m - 1$.
Let $A = \{1, 3, ..., 4mn + 2m - 1\}$.
Define a vertex labeling $f : V(T\widetilde{O}QS_n) \to A$ as follows:

For $1 \leq i \leq m, 1 \leq j \leq n+1$ $f(u_{ij}) = \begin{cases} (4n+2)(i-1) + 4j - 3 & \text{if } i \text{ is odd} \\ (4n+2)i - (4j-3) & \text{if } i \text{ is even} \end{cases}$.

For $1 \leq i \leq m, 1 \leq j \leq n$.
$f(v_{ij}) = f(u_{ij})$, $f(w_{ij}) = \begin{cases} (4n+2)(i-1) + 4j - 1 & \text{if } i \text{ is odd} \\ (4n+2)i - (4j-1) & \text{if } i \text{ is even} \end{cases}$.
$f(u_i^{'}) = \begin{cases} (4n+2)i - 1 & \text{if } i \text{ is odd} \\ (4n+2)i - (4n+1) & \text{if } i \text{ is even} \end{cases}$.

For the vertex labeling $f$, the induced edge labeling $f^*$ is as follows:

For $1 \leq i \leq m-1$ $f^*(e_i^{'}) = 2(4n+2)i$,

For $1 \leq i \leq m$ $f^*(e_i^{''}) = \begin{cases} 2(4n+2)i - 2 & \text{if } i \text{ is odd} \\ 2(4n+2)(i-1) + 2 & \text{if } i \text{ is even} \end{cases}$.

The induced edge labels of $QS_n$ are $2(4n+2)(i-1)+2j(1 \leq i \leq m, 1 \leq j \leq 2n)$ if $i$ is odd and $2(4n+2)(i-1)+2j(1 \leq i \leq m, 1 \leq j \leq 2n)$ if $i$ is even.

Let $v_iv_j$ be a transformed edge in $T$ for some indices $i, j, 1 \leq i \leq j \leq m$.

Let $P_1$ be the ept that deletes the edge $v_iv_j$ and adds an edge $v_{i+t}v_{j-t}$ where $t$ is the distance of $v_i$ from $v_{i+t}$ and the distance of $v_j$ from $v_{j-t}$.

Let $P$ be a parallel transformation of $T$ that contains $P_1$ as one of the constituent epts. Since $v_{i+t}v_{j-t}$ is an edge in the path $P(T)$, it follows that $i + t + 1 = j - t$ which implies $j = i + 2t + 1$.

Therefore, $i$ and $j$ are of opposite parity, that is, $i$ is odd and $j$ is even or vice-versa.

The induced label of the edge $v_iv_j$ is given by $f^*(v_iv_j) = f^*(v_iv_{i+2t+1}) = f(v_i) + f(v_{i+2t+1}) = 2(4n+2)(i+t)$ and

$f^*(v_{i+t}v_{j-t}) = f^*(v_{i+t}v_{i+t+1}) = f(v_{i+t}) + f(v_iv_{i+t+1}) = 2(4n+2)(i+t).$

*Therefore,* $f^*(v_iv_j) = f^*(v_{i+t}v_{j-t}).$

*It can be verified that the induced edge labels of* $T\widetilde{O}QS_n$ *are* $2, 4, 6, ..., 8mn + 4m - 2$ *and* $|v_f(a) - v_f(b)| \leq 1$ *for all* $a, b \in A$.

*Hence,* $T\widetilde{O}QS_n$ *is an odd vertex equitable even graph.*

*An example for odd vertex equitable even labeling of* $T\widetilde{O}QS_2$ *where* $T$ *is a* $T_p$ *-tree on 8 vertices is shown in Figure 2.4.*

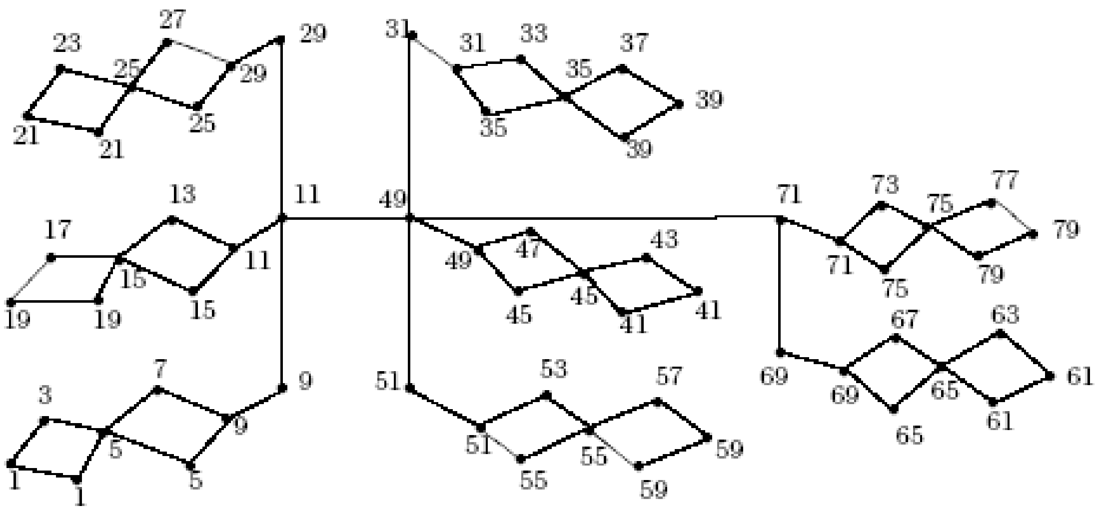


Figure 2.4

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## References


[1] *C. Barrientos, Difference Vertex Labelings, Ph.D. Thesis, Universitat Politecnica de Catalunya, Barcelona, (2004).*

[2] *Joseph A. Gallian, A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics, 19, (2017)#DS6.*

[3] *F. Harary, Graph theory, Addison Wesley, Massachusetts, (1972).*

[4] S. M. Hedge and Sudhakar Shetty, On Graceful Trees, *Applied Mathematics E-Notes*, **2**, *pp. 192-197, (2002).*

[5] P. Jeyanthi and A. Maheswari, Some results on vertex equitable labeling, *Open J. Discrete Math.*, **2**, *pp. 51-57, (2012).*

[6] P. Jeyanthi and A. Maheswari, Vertex equitable labeling of Transformed Trees, *Journal of Algorithms and Computation*, **44**, *pp. 9-20, (2013).*

[7] P. Jeyanthi and A. Maheswari, Vertex equitable labeling of cyclic snakes and bistar graphs, *Journal of Scientific Research*, **6**, *(1), pp. 79-85, (2014).*

[8] P. Jeyanthi and A. Maheswari, Vertex equitable labeling of families of graphs, *Graph theory Notes of New York* , **LXVII**, *pp. 34-42, (2014).*

[9] P.Jeyanthi and A. Maheswari, Vertex Equitable Labeling of Super Subdivision Graphs, *Scientific International*, **27** *(4), pp. 1-3, (2015).*

[10] P. Jeyanthi, A. Maheswari and M. Vijayalakshmi, Vertex equitable labeling of cycle and star related graphs, *Journal of Scientifc Research*, **7**, *pp. 33-42, (2015).*

[11] P. Jeyanthi and A. Maheswari, Vertex equitable labeling of cycle and path related graphs, *Utilitas Mathematica,* **98**, *pp. 215-226, (2015).*

[12] P. Jeyanthi, A. Maheswari and M. Vijaya Lakshmi, Vertex equitable labeling of Double Alternate Snake Graphs, *Journal of Algorithms and Computation*, **46**, *pp. 27-34, (2015).*

[13] P. Jeyanthi, A. Maheswari and M. Vijaya Lakshmi, *New Results on Vertex Equitable Labeling, Journal of Algebra Combinatorics Discrete structures and Applications*, **3**, (2), pp. 97-104, (2016).

[14] P.Jeyanthi, A. Maheswari and M. Vijaya Lakshmi, *Vertex Equitable Labeling of Union of Cyclic Snake graphs, Proyecciones Journal of Mathematics*, **35**, (2), pp. 177-186, (2016).

[15] P. Jeyanthi, A. Maheswari and M. Vijaya Lakshmi, *Odd Vertex Equitable Even Labeling, Proyecciones Journal of Mathematics*, **36**, (1), pp. 1-11, (2017).

[16] A. Lourdusamy and M. Seenivasan, *Vertex equitable labeling of graphs, Journal of Discrete Mathematical Sciences and Cryptography*, **11**, (6), pp. 727-735, (2008).

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